

DEVELOPMENT OF A SIMPLIFIED CAT ALGORITHM, INVOLVING BESSEL FUNCTIONS

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CERTIFICATE

This is to certify that this work on "Development of a Simplified CAT Algorithm, involving Filtered Bessel Functions" by Major V.K. Bhatia has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.

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-Major V.K. Bhatia

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ABSTRACT

Computer Aided Tomography (CAT) is widely used in the medical imaging area for the detection of cancerous tissues. It has also proved to be a good technique for measuring point-density (void-fraction) in two-phase flow systems. The present day CAT-scanners employ algorithms which are essentially a discrete implementation of the Radon Inversion Formulae. These methods of reconstructing density distributions are fast and easy to implement.

The present work is an attempt to simplify (and consequently to speed-up) the existing algorithms for the cases of radially symmetric density distribution. The method uses Radon Inversion Formulae and involves Bessel's Function of Fourier frequency.

The algorithm was tested on some known distributions and the reconstruction is excellent. Data for bubbly-flow was also processed to obtain a point density distribution. The range of investigation was $0.6 \leq \langle \rho \rangle \leq 0.9 \text{ gm/cm}^3$, where " $\langle \rangle$ " denotes average cross-sectional value. The values of $\langle \rho_{CT} \rangle$ are within $\pm 0.03 \text{ g/cm}^3$ of the true values, $\langle \rho \rangle$, for the data sets considered. The results are comparable with another simple method involving radial polynomials. It is observed that the simplified Radon Inversion Formulae for the radially symmetric case gives good results and can be implemented with ease.

CHAPTER 1

INTRODUCTION

The problem of image reconstruction from projections has repeatedly arisen over the last four decades in a large number of scientific, medical and technical fields. Of all the applications, probably the greatest effect on the world at large has been in the area of diagnostic medicine. Computer-Aided-Tomography (CAT) has revolutionized the techniques in radiology.

Images of cross-sections of the human body are produced from the data obtained by measuring the attenuation of radiation along a large number of lines through the cross-section. The CAT machine incorporates scanning of patient with radiation using appropriate tomographic algorithm to reconstruct the density distribution of tissues [1].

A major problem confronting an engineer in the Nuclear reactor safety field is the measurement of void fraction (or density) during various accident/transient situations. The development of an accurate and reliable two-phase flow measurement technique for various flow systems and components in Light Water Reactor (LWR) is of great importance and significance. The concept of measuring density distribution was first investigated by Schlosser et al [2] for a two-phase air-water system. The density measurement

results have been summarized by Kulacki et al [3].

In many applications where images are required, the measurements can be done by probing the object by invisible radiations and interpreting the measured data. Sometimes immediate interpretation of measured data is not possible directly, but is related to the image in a known way. Generally, all image reconstruction procedures or algorithms aim at processing the data to form the image and thereby to facilitate the interpretation of measurement.

The present work mainly looks at such a new field, which will alleviate the problems (related to radially symmetric distribution) in the nuclear engineering area (two-phase flow in reactor systems), chemical industry, food processing etc.

The most important feature of tomographic reconstruction method is its non-invasive aspect of point-density measurement. Due to this fact, the system retains its natural/original configuration, without being even slightly affected by the measuring device.

The present work is aimed at developing a simplified algorithm based on the Radon Inversion Formulae, involving Bessel Functions, which can perform tomographic reconstruction of the density distribution for radially symmetric two-phase flows. The data is collected in the parallel-beam mode.

The algorithm written in FORTRAN, has been initially tested against some known functions, simulated radially symmetric distributions, and annular flows. The algorithm has also been successfully implemented to reconstruct the density map for air-water bubbly flow data [3,4] . The results are comparable with the results obtained by the other known general methods [3,4,5] . The results are also comparable with another alternate scheme involving radial polynomials [6,7] .

CHAPTER 2

THEORETICAL FORMULATION

This chapter includes a brief discussion of:

- (a) methods of data collection in a tomographic system depending upon geometry and,
- (b) the relation of this data to the distribution.

Two standard geometries of data collection, namely, fan beam and parallel beam geometries are discussed. In addition, types of algorithms, two phase flow and use of CAT in identifying flow regimes are briefly explained.

2.1 Data Collection

In a tomographic system, **radiation** is passed through the object (to be reconstructed) and the attenuated signal is detected. This signal is related to the function to be reconstructed. The single beam radiation attenuation phenomenon is represented by:

$$N = N_0 \exp \left\{ - \int_0^S \mu(r, \theta) ds \right\} \quad (1)$$

where,

N = detector reading (counts/s)

N_0 = source strength (counts/s)

S = path of radiation (ray)

C = chord along which S is integrated

μ = absorption coefficient

r, ϕ = cylindrical co-ordinates.

Rewriting eqn.(1), we have

$$p = \int_C \mu(r, \phi) ds \quad (2)$$

where

$$p = \ln (N_0/N) \quad (3)$$

2.2 Parallel Beam Geometry Data Collection

A line in the plane is specified by two parameters; its (signed) distance l from the origin and its angle θ with respect to the Y-axis (Fig.1). The function of two variables, whose value for any (l, θ) is defined as line-integral of μ along the line specified by l and θ , is denoted by $p(l, \theta)$. For any l and θ , (l, θ) and $(-l, \theta + \pi)$ represents the same line in the plane, so that

$$p(l, \theta) = p(-l, \theta + \pi) \quad (4)$$

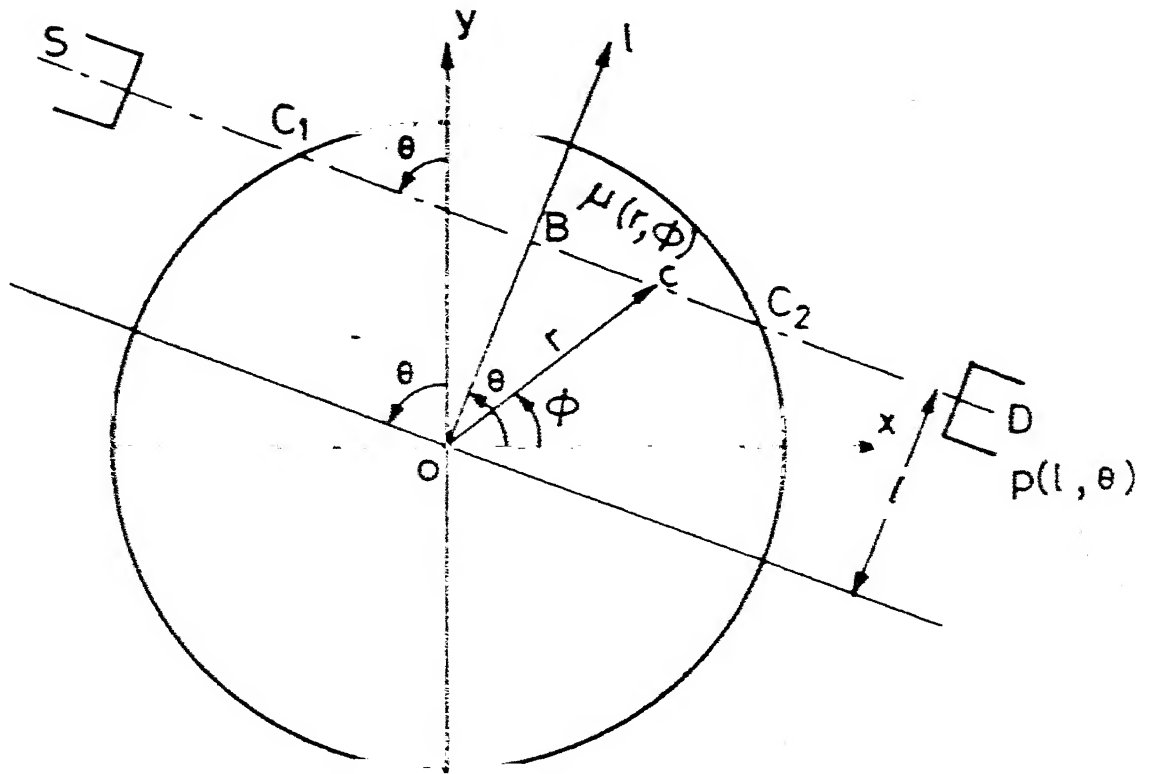
By projection theorem [8] we know that

$$\hat{\mu}(R \cos \theta, R \sin \theta) = \hat{p}(R, \theta) \quad (5)$$

where

$$\hat{p}(R, \theta) = \int_{-D}^D p(l, \theta) e^{-i2 R l \pi} dl \quad (6)$$

PARALLEL BEAM GEOMETRY



LEGEND: S - Source
D - Detector

Points within object are described by a fixed (x, y) or (r, ϕ) co-ordinate system. Rays (dashed-line) are specified by their angle with respect to y axis, θ and their distance from origin l .

Fig.1 Data collection geometry

where

D is the object radius and

R corresponds to the Fourier frequency.

The Radon Inversion Formulae [1] relates the data $p(l, \theta)$ to the function $\mu(r, \phi)$ as follows:

$$\mu(r, \phi) = \int_0^\pi \int_{-\infty}^{\infty} \hat{p}(R, \theta) e^{i2\pi R \cos(\theta - \phi)} |R| dR d\theta \quad (7)$$

Considering the radially symmetric flow into account; μ no longer remains a function dependent on ϕ , so that we can choose any particular ϕ and obtain the reconstructed $\mu(r)$. For computational ease, and without loss of generality, we choose $\phi = 0$. Thus eqn.(7) becomes,

$$\mu(r) = \int_0^\pi \int_{-\infty}^{\infty} \hat{p}(R) e^{i2\pi R \cos\theta} |R| dR d\theta \quad (8)$$

where

$$\hat{p}(R) = \int_{-D}^D p(l) e^{-i2\pi R l} dl \quad (9)$$

We see that $p(l, \theta)$ has been replaced by $p(l)$ since radial symmetry in μ implies the independence of p with respect to the source position, θ .

Interchanging order of integration in eqn.(8), we get

$$\mu(r) = \int_{-\infty}^{\infty} \hat{p}(R) \left(\int_0^\pi e^{i2\pi R \cos\theta} d\theta \right) |R| dR$$

Here integral over Θ is a Bessel Function [9], so that the inversion formula takes a particularly simple form,

$$\mu(r) = 2\pi \int_0^{\infty} \hat{p}(R) J_0(2\pi r R) |R| dR \quad (10)$$

A practical implementation would require a finite cut-off for the variable, R , the Fourier frequency. Denoting this cut-off by R_c , we get

$$\mu(r) \approx 2\pi \int_0^{R_c} \hat{p}(R) J_0(2\pi r R) |R| dR \quad (11)$$

Keeping in mind the discrete nature of the data $p(l)$, a reasonable choice of R_c , based on sampling theorem [10], is given by

$$R_c = 1/2 \Delta l$$

where, Δl is spacing between the rays. It is to be noted that the expression given in eqn.(11) is similar to the point-spread function expression reported by Lewitt [8].

In the design of reconstruction algorithm using Transform approach, it is often necessary to introduce a window function $W(R)$:

$$W(R) = \begin{cases} W(R) & |R| < R_c \\ 0 & |R| \geq R_c \end{cases}$$

giving eqn.(11), the following form:

$$\mu(r) \approx 2\pi \int_0^{R_c} \hat{p}(R) J_0(2\pi r R) |R| W(R) dR \quad (12)$$

Two different types of window functions have been tested, viz., linear and sinc functions. A sensitivity study has also been performed on the slope of the linear window function. The forms used are:

Linear

$$W(R) = 1 - \frac{\epsilon R}{R_c}$$

where

$$0 \leq \epsilon \leq 1.0$$

Sinc

$$W(R) = \frac{\text{Sin}(\pi R/R_c)}{(\pi R/R_c)}$$

2.3 Fan Beam Geometry

In this case, one needs a rotating source and a ring of stationary detectors on the outer periphery. Scanning involves only motion of the source.

In medical applications of CT, in order to reduce distortion due to movement of patient, data should be collected in shortest possible time. This is facilitated by

using fan beam geometry of Radiation diverging from a source, passing through the patient and intercepted by detector elements. From the transmitted radiation, intensities are recorded while source moves in a circular path around the patient.

The relationship between both the geometries [8], is as follows:

$$l = D \sin \sigma, \quad \theta = \beta + \sigma$$

$$p(l, \theta) = g(\sigma, \beta) .$$

2.4 Types of Algorithms

For image reconstruction from projection data, the algorithms may be categorised as under:

- (a) Transform Methods [8]
- (b) Series Expansion Method [11] and
- (c) Distributional Approximation Method .

(a) Transform Methods:

Based on Radon Inversion Technique, the Transform methods basically rely upon analytic formulae . Transform methods [8] are broadly categorized as under:

- (i) Direct Fourier Inversion (DFI): In this method, direct Fourier Transform of projection data is taken. Subsequently 2-d Fourier Inversion

leads to the reconstruction of unknown distributions.

- (ii) Convolution Back Projection (CBP): This method is based on convolving data with a suitable filter function. The back projection of this convolved data results in the reconstruction of unknown distribution.

(b) Series Expansion Method:

In this method pixel-wise distribution of the function under-investigation is assumed. To obtain reconstruction of function in the region of interest [11], suitable iterative and non-iterative methods are applied.

The iterative SEMs are:

- (i) Algebraic Reconstruction Technique and
- (ii) Simultaneous Iterative Reconstruction Technique etc.

Whereas non-iterative SEMs are:

- (i) Angular Harmonic Decomposition and
- (ii) Polynomial Decomposition.

(c) Distributional Approximation Method:

The distributional approximation methods developed at IIT Kanpur involve assuming the distribution on the entire region of interest.

The RAP method [6,7] incorporates radial polynomials as the basis functions while the CSI Method [12] involves constant ring-values as the basis functions in its formulation.

The approach used in the present study involves the following steps:

- (1) Read data p in the form of a column vector corresponding to the spacing between rays.
- (2) Computation of $p(R)$.
- (3) Computation of μ -values along the chord using eqns. (11,12) (involving Bessel Function).
- (4) Calibration of μ -values to the density values.

For the validation studies, data was generated (eqn.(2)), whereas in the experimental air-water flow studies, data is available from the detector counts. For computation of $p(R)$, a simple integration scheme was adopted (Appendix E).

2.5 Two-Phase Flow

Some of the major problems in the nuclear industry are:

- (a) Measurement of mass-flow rate of two-phase flow,
- (b) Finding nature of flow-around fuel bundle in Light Water Reactor (LWR),

- (c) Developing reliable two-phase flow measuring technique for various flow systems in a reactor.

Earlier photographic methods for identification of flow regimes, using appropriate correlation to measure void-fraction were used. With the development of CAT, problem of determination of mass flow rate and identification of flow regimes has been simplified.

Different two-phase flow regimes, namely, bubbly, slug, annular and dispersed flows can be investigated for determining void-fractions. However, to determine mass-flow rate, it is essential to know void-fraction without disturbing the flow field. This can be achieved by Tomographic Methods much more accurately than by other existing methods.

CHAPTER 3

RESULTS

3.1 Validation Against Simulated Data

We discuss the results for the simulated data in this section. We assume the object size/pipe radius to be one unit and the ray spacing 0.1.

The algorithm was tested against the simulated data on the following symmetric distributions:

$$\mu(r) = 1.0$$

$$\mu(r) = r$$

$$\mu(r) = \exp(r) \text{ and}$$

$$\mu(r) = \exp(-r)$$

The reconstruction is within $\pm 2\%$ for all the test functions except in the peripheral region. The true and reconstructed values are shown in Fig.2 .

The algorithm has also been tested for annular flows, having distributions as under:

$$\mu(r) = \begin{cases} 1.0 & 0 \leq r < 0.4 \\ 0.0 & 0.4 \leq r \leq 1.0 \end{cases}$$

$$\mu(r) = \begin{cases} 0.0 & 0 \leq r < 0.2 \\ 1.0 & 0.2 \leq r \leq 0.4 \\ 0.0 & 0.4 \leq r \leq 1.0 \end{cases}$$

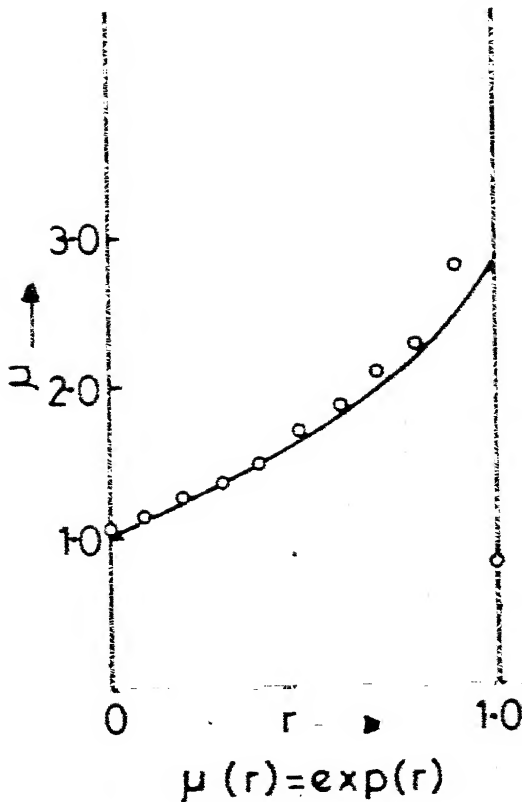
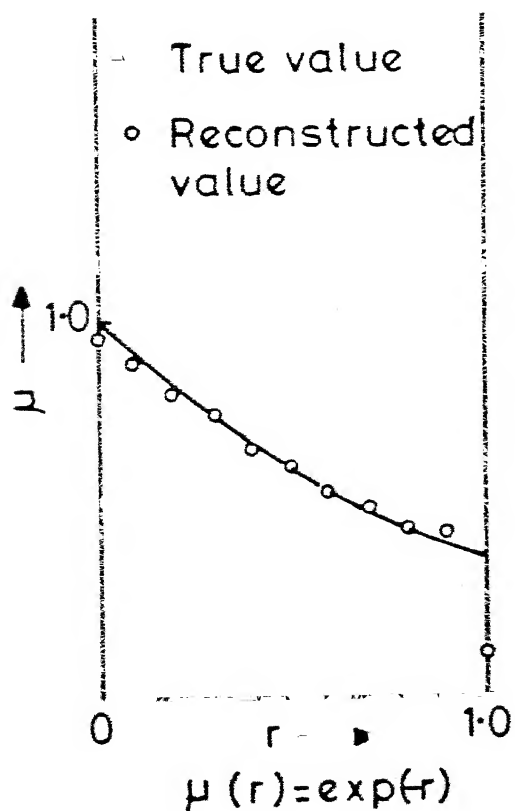
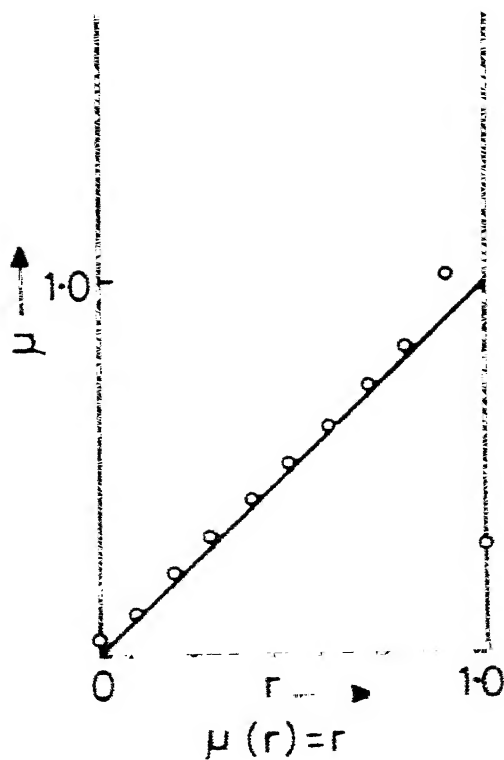
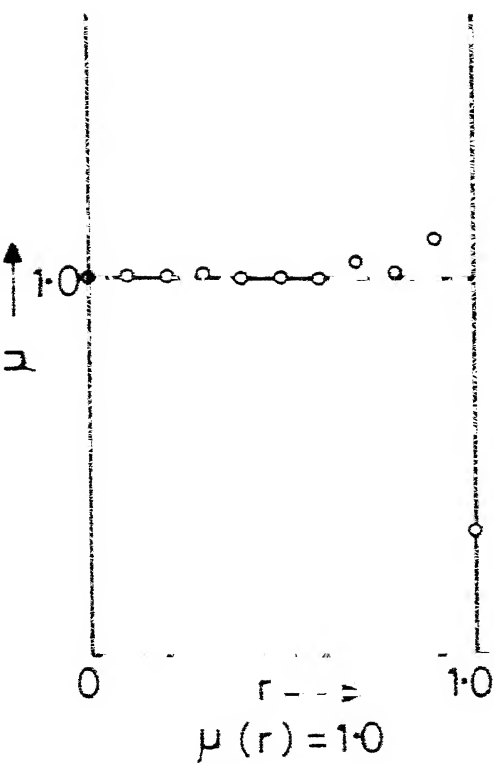


Fig.2 Reconstructed results for test-functions

In a calibrated sense, the above mentioned distributions represent the various types of density/void fraction distributions encountered in radially symmetric bubbly and annular flows. The reconstruction is shown in Figs. 3 and 4. The solid lines represent the assumed functions and the dotted lines indicate the reconstructed values for the unfiltered and filtered versions.

Testing of algorithm for Dirac Delta (δ) function (at the center of the pipe) was also carried out. Both the filters viz. linear and sinc were used for δ -function. The results are encouraging, both for unfiltered and filtered versions (See Fig. 5).

3.2 Results for Bubbly Air Water-Flows

The test data is taken from the study of Ref. [3,4]. Five different data sets for four different densities/void fractions were processed by this algorithm. The algorithm output, CTN, was calibrated to obtain the density. To do this, the data was processed for the following four known densities:

$$\begin{aligned}\text{Air, } \langle \rho \rangle &= 0.00 \text{ g/cm}^3, \\ \text{Pine, } \langle \rho \rangle &= 0.41 \text{ g/cm}^3, \\ \text{Walnut, } \langle \rho \rangle &= 0.732 \text{ g/cm}^3, \\ \text{Water, } \langle \rho \rangle &= 1.00 \text{ g/cm}^3,\end{aligned}$$

where " $\langle \rangle$ " denotes the cross-sectionally averaged values.

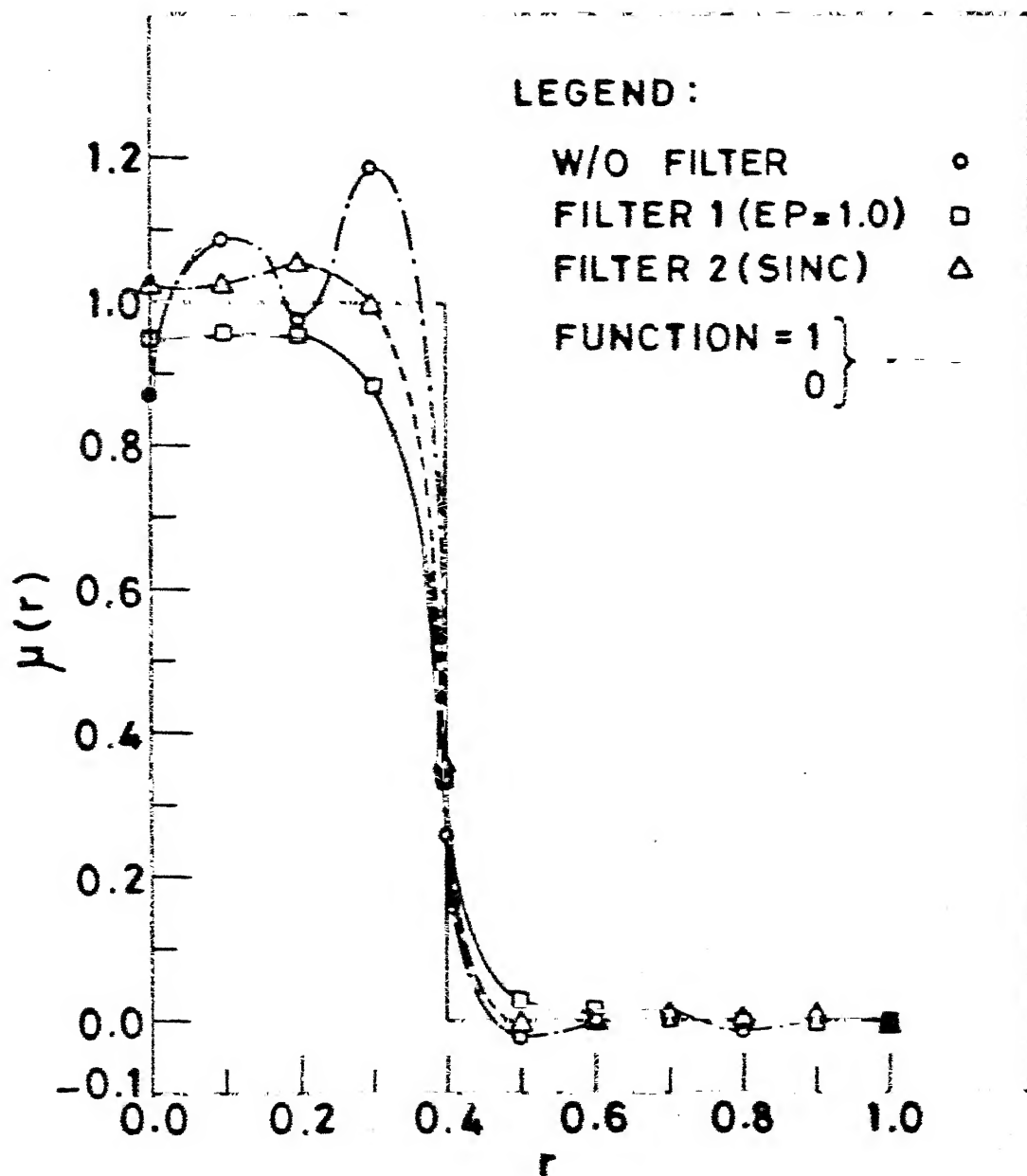


FIG. 3 RESULTS WITH FILTERED VERSION (ANNULAR FLOW).

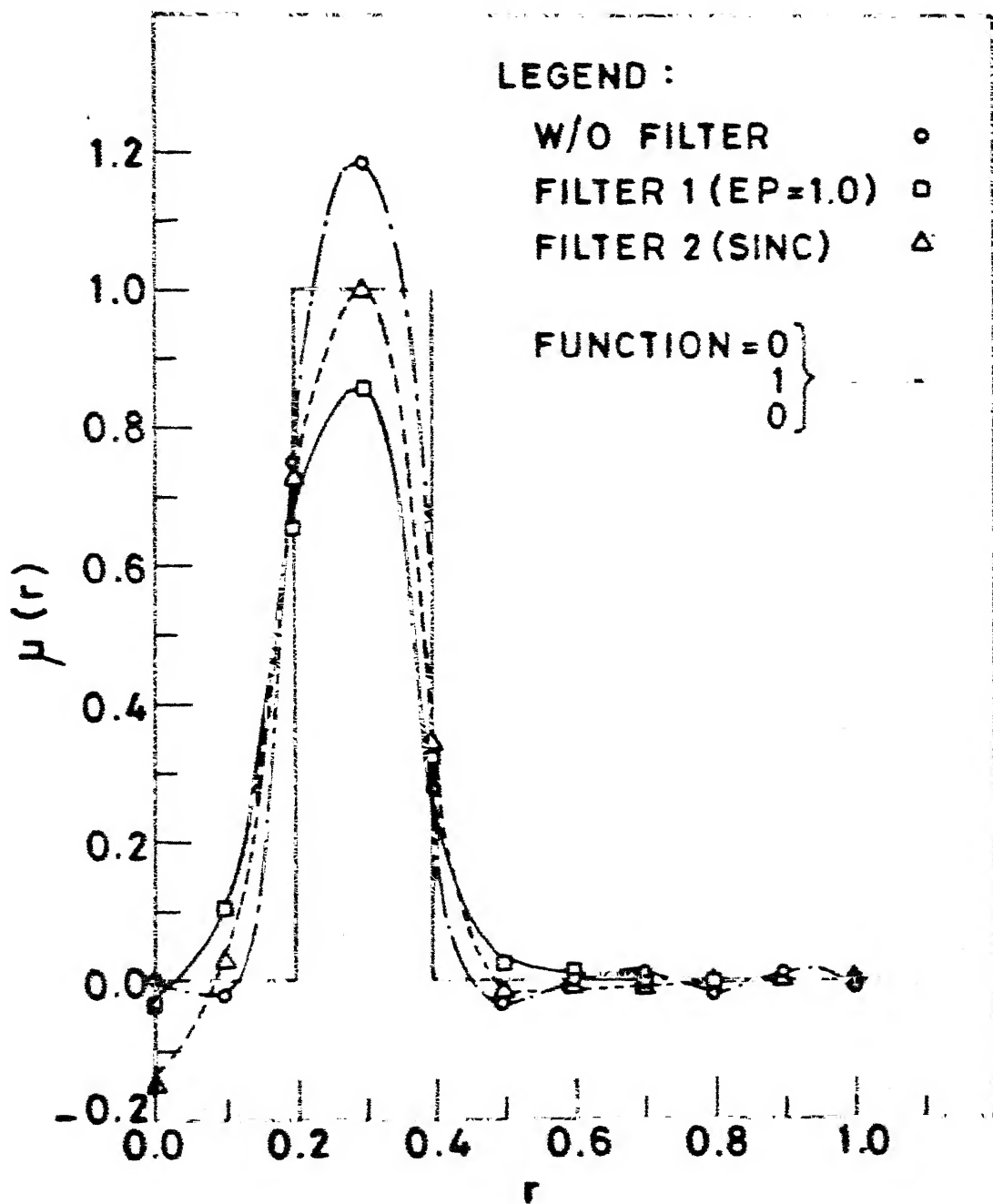


FIG. 4 RESULTS WITH FILTERED VERSION (ANNULAR FLOW).

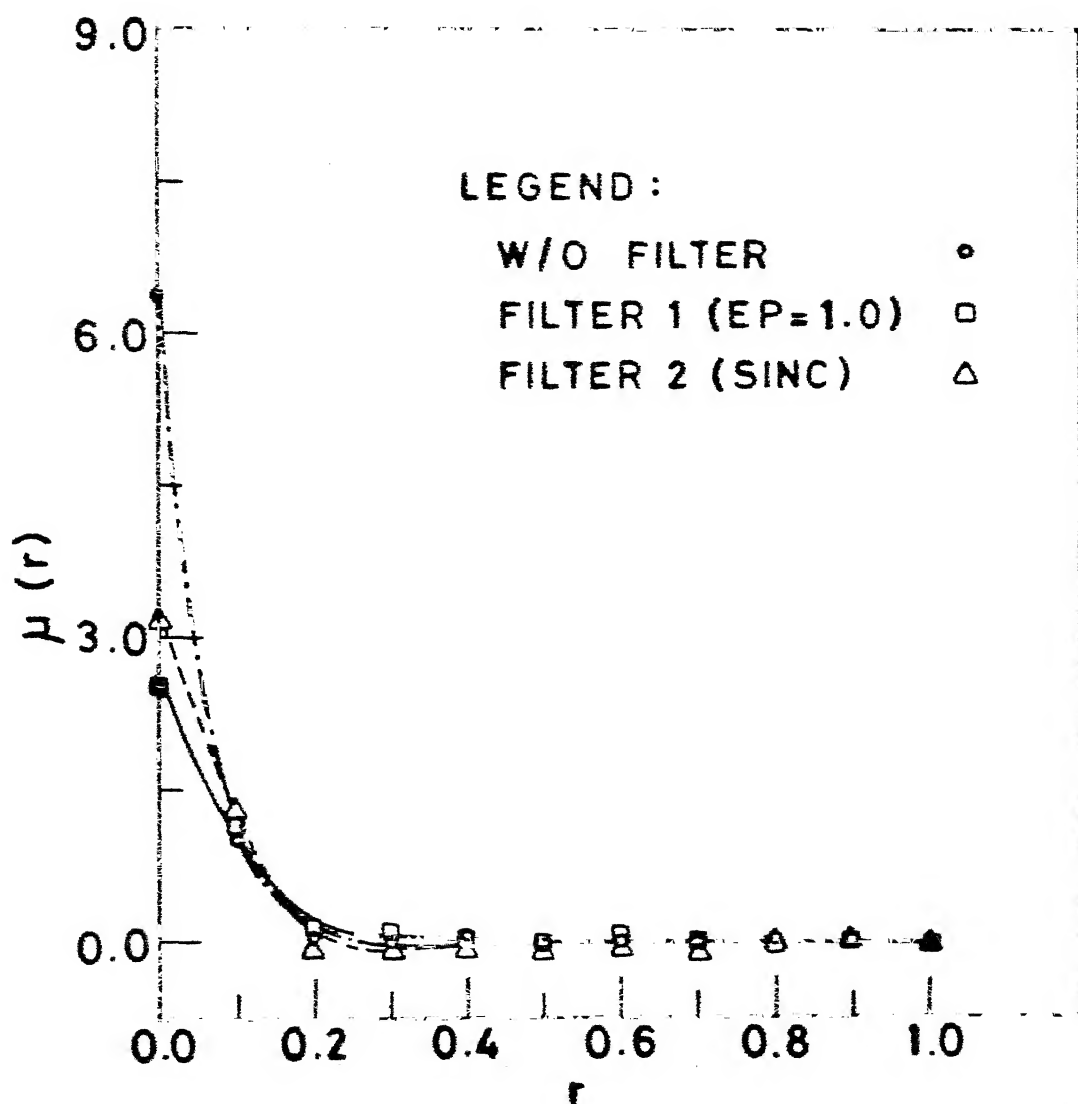


FIG.5 RESULTS WITH FILTERED VERSION
(DELTA FUNCTION).

The output CTN, was corrected to the base of air ($\text{CTN} = 0.0$) to eliminate the effect of the pipe-wall, made of plexiglass. We know, for air, the absorption coefficient (CTN) must be zero, whereas for air data set, we get non-zero values for the $\langle \text{CTNs} \rangle$. This effect is due to attenuation in the plexiglass pipe. In order to get the correct CTNs for all cases, we subtract the CTNs for the air ($\rho = 0.0 \text{ g/cm}^3$) case from the reconstructed values. This way, we can get correct CTNs and $\langle \text{CTNs} \rangle$ for any case. The CTNs obtained are listed in Appendix B. The calibration curve [$\langle \rho \rangle$ vs. $\langle \text{CTN} \rangle$] is shown in Fig. 6, by which, for any known $\langle \text{CTN} \rangle$, the corresponding density value can be obtained.

Similarly data for different scans were processed and values of CTNs for the 12 rings in the pipe cross-section were achieved. For each radius, the ρ value obtained after calibration, have been plotted. Figures 7-11 show the reconstructed density profiles for various densities, $\langle \rho \rangle$, (or void-fractions $\langle \alpha \rangle$) for all the scans.

In Fig. 12, the comparison of reconstructed densities with actual densities (measured by an alternate method [3,4]) have been shown. The various densities obtained by calibration method are marked to show the deviation of the reconstructed densities from the actual densities. The summary of $\langle \text{CTN} \rangle$ and $\langle \rho \rangle$ for all scans is given in

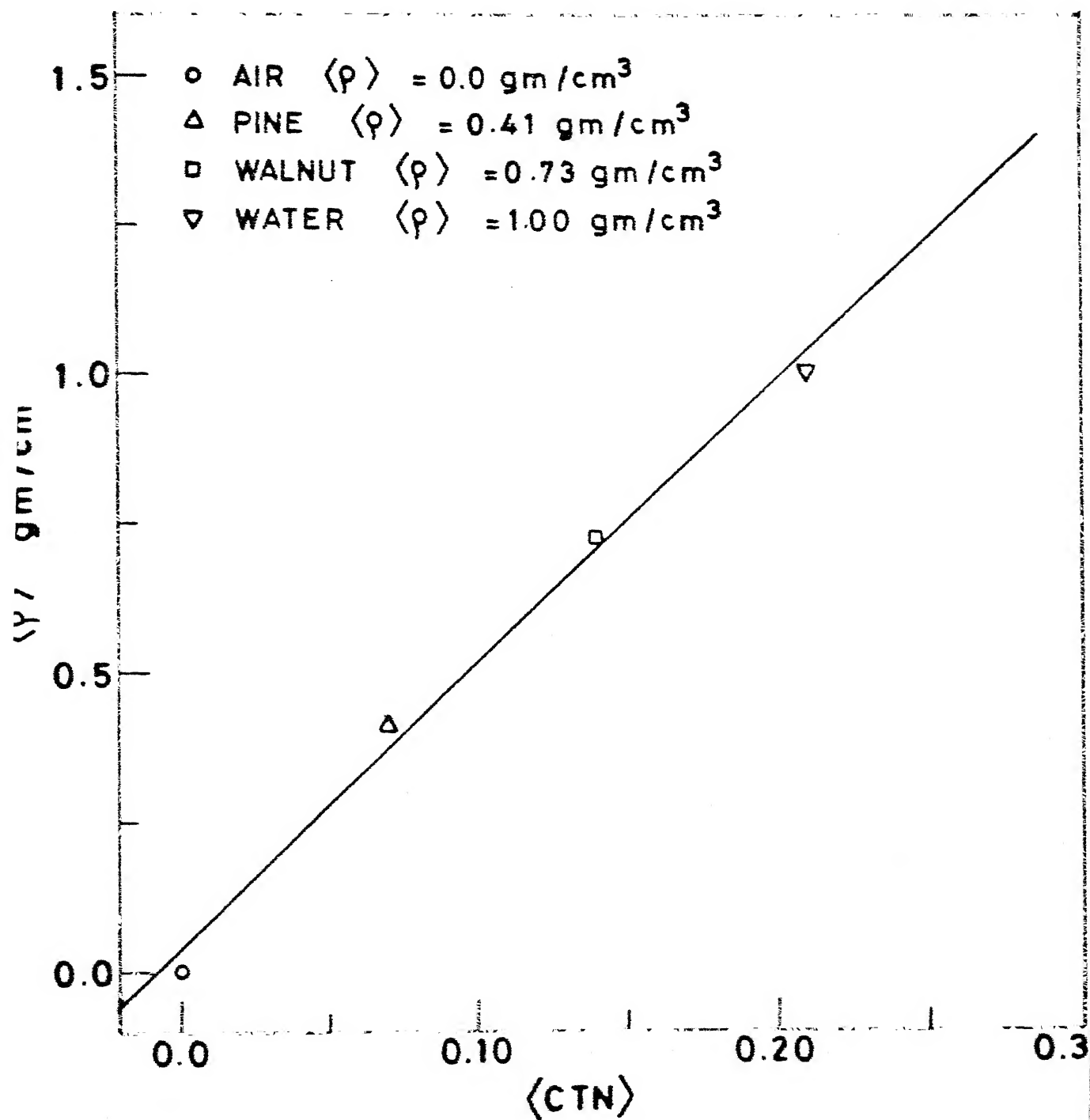


FIG. 6 CALIBRATION CURVE

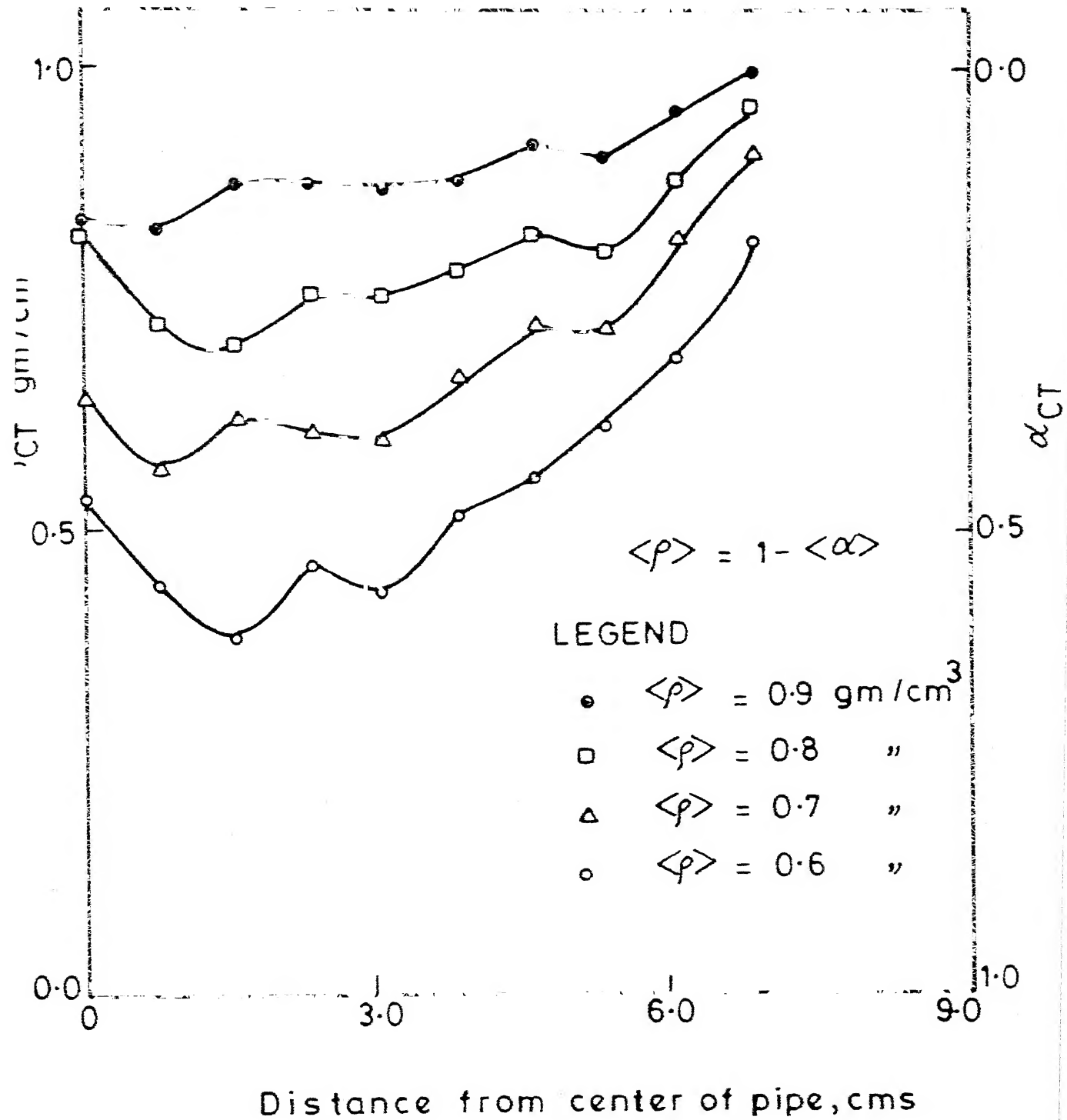


Fig.7 Reconstructed density profile(Radial)
SCAN 1.

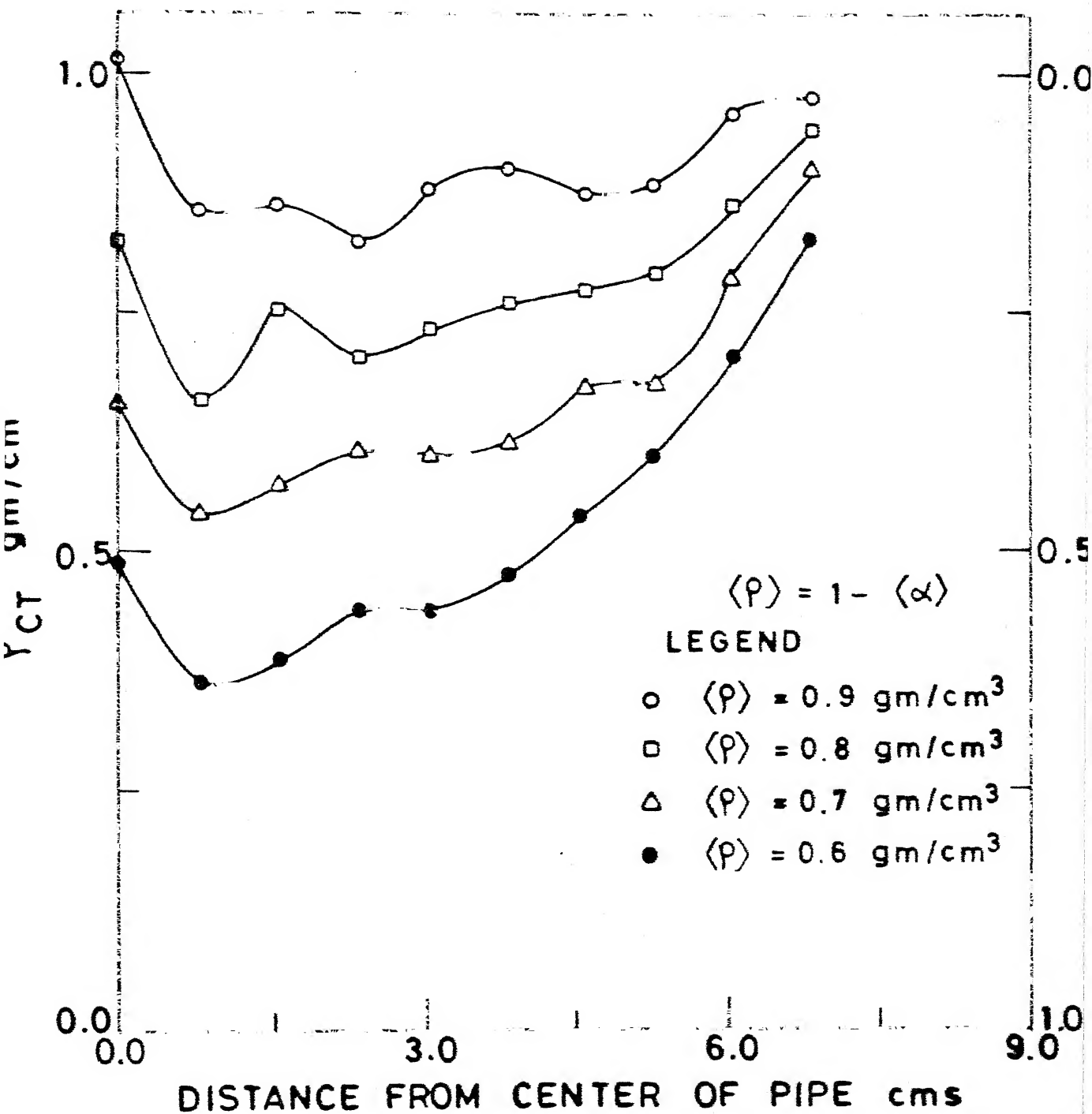


FIG. 8 RECONSTRUCTED DENSITY PROFILE (RAD SCAN 2).

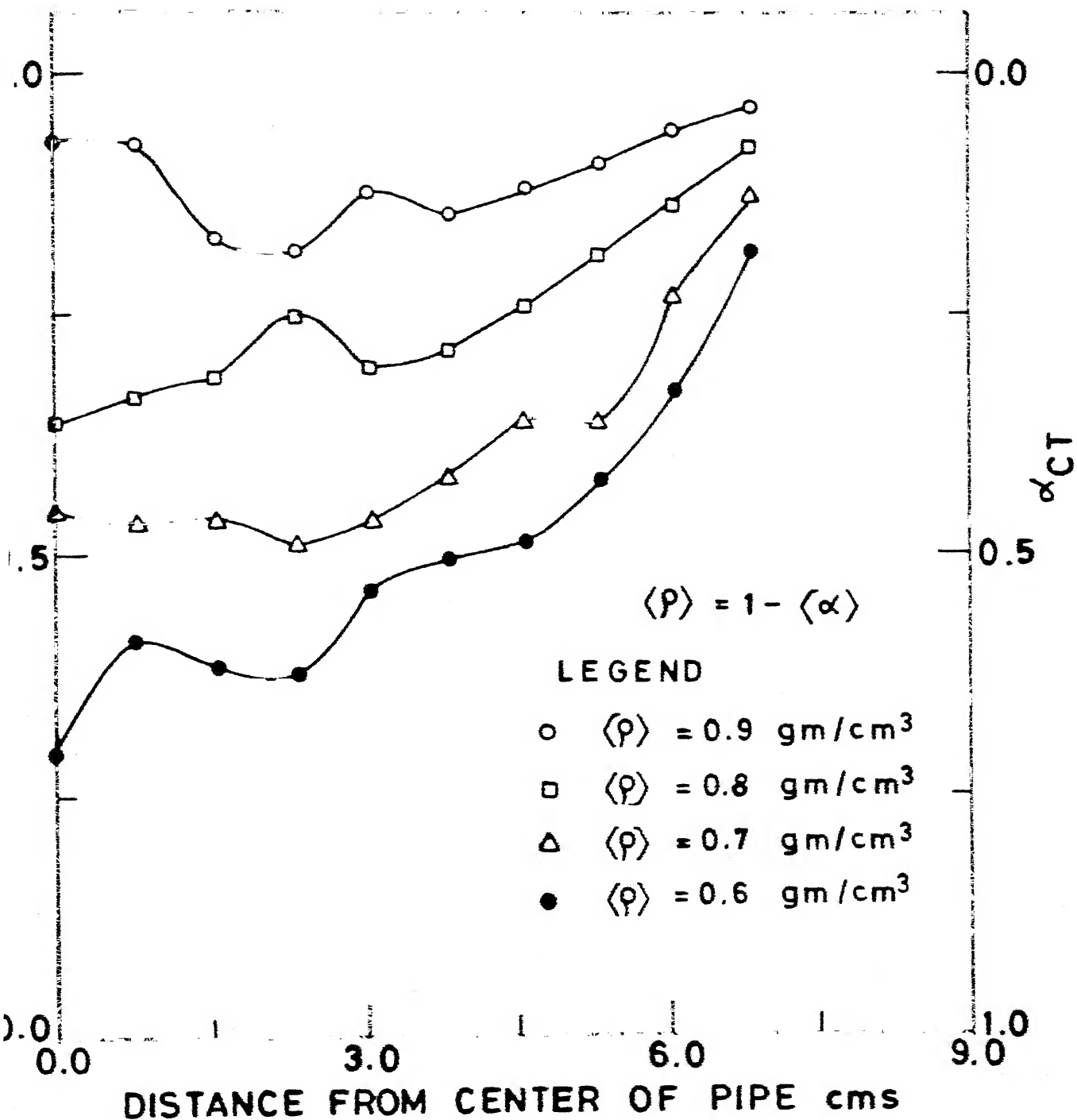


FIG. 9 RECONSTRUCTED DENSITY PROFILE (RADIAL) SCAN 3 .

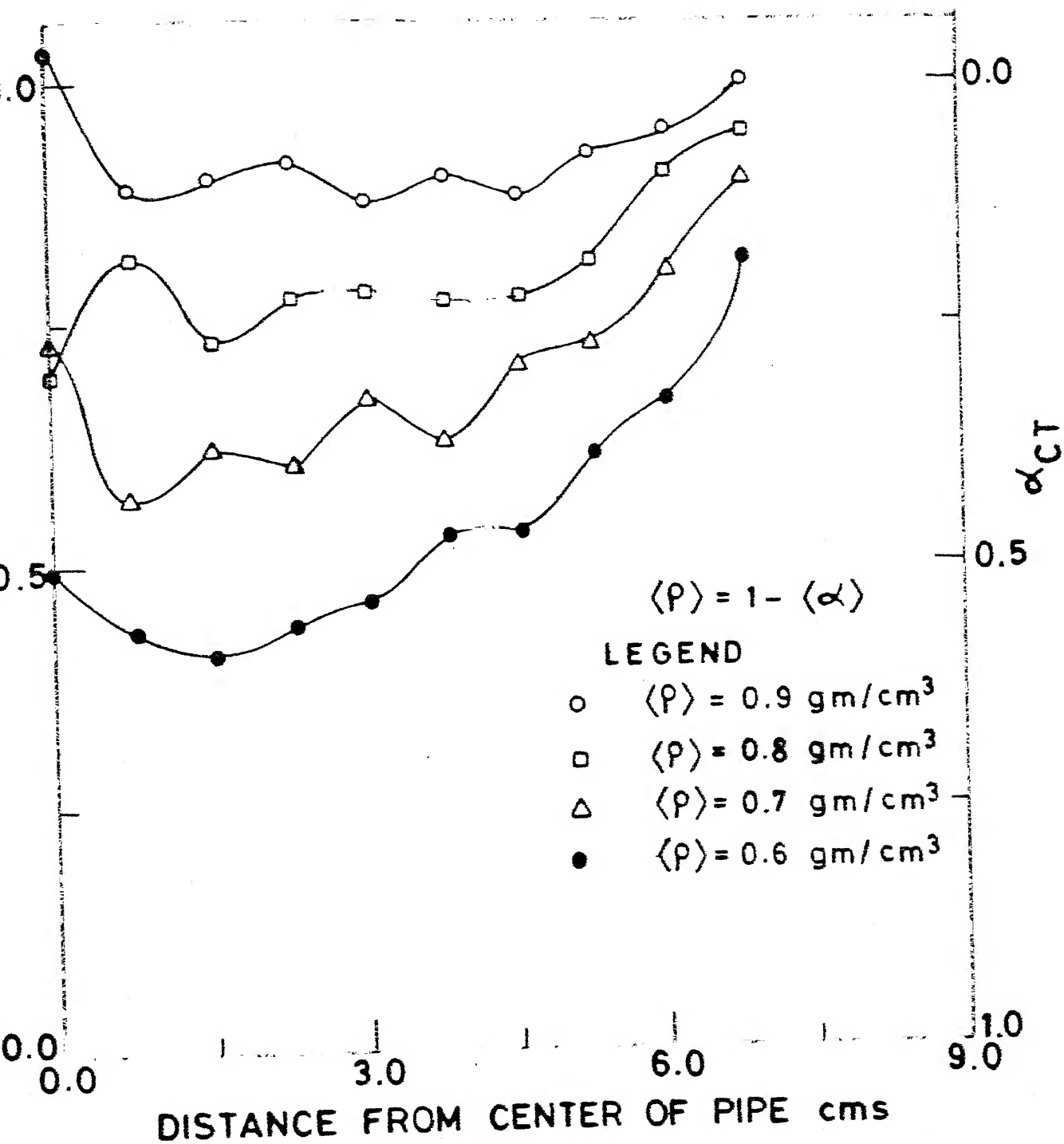


FIG.11 RECONSTRUCTED DENSITY PROFILE (RADIAL)
SCAN 5.

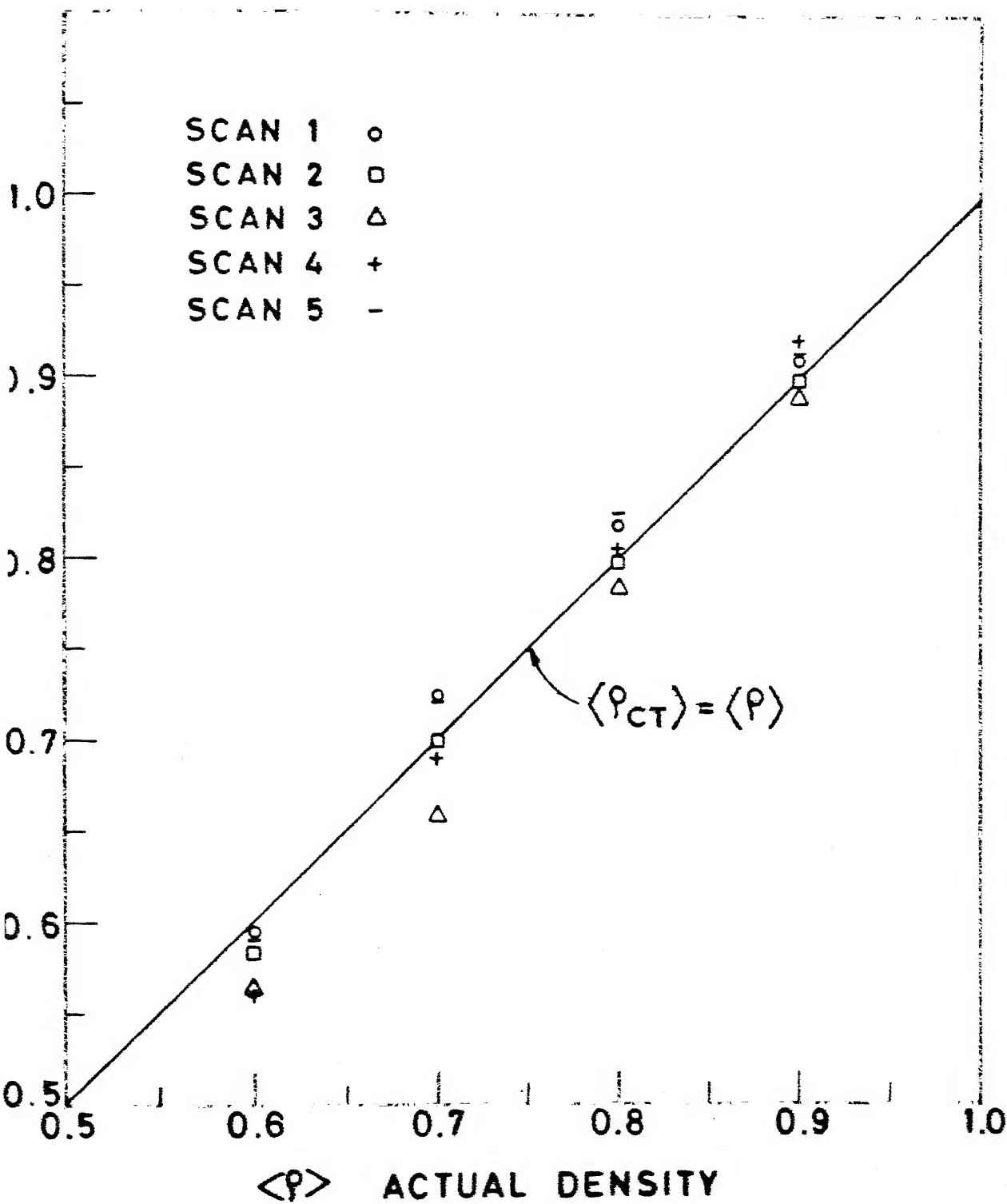


FIG.12 COMPARISON OF RECONSTRUCTED DENSITY WITH AN ALTERNATE METHOD.

Appendix B.

It is observed that the reconstruction is within $\pm 0.03 \text{ g/cm}^3$ of the actual values. The variation of error with density has been plotted in Fig.13. The variation of relative error with respect to density is shown in Fig. 14 and it ranges between $+ 3.57\%$ to -6.67% .

3.3 Results with Filtered Version:

The two filters viz. linear and sinc, applied to four known cases of distributions of $1, r, \exp(r)$ and $\exp(-r)$, give encouraging results. Figures 15-18 show the distributions without filter and with these two filters separately, for these four known cases. The deviation of the reconstructed values from the true values is within $\pm 0.03 \text{ g/cm}^3$, (except for the peripheral region) for all the filters tested in the study. It is to be noted that near the edges the density falls off sharply and a reconstruction incorporating a continuous function (Bessel function in this case) naturally leads to discrepancies in the peripheral region. However such errors do not significantly effect the reconstruction quality of the overall cross-sectional image.

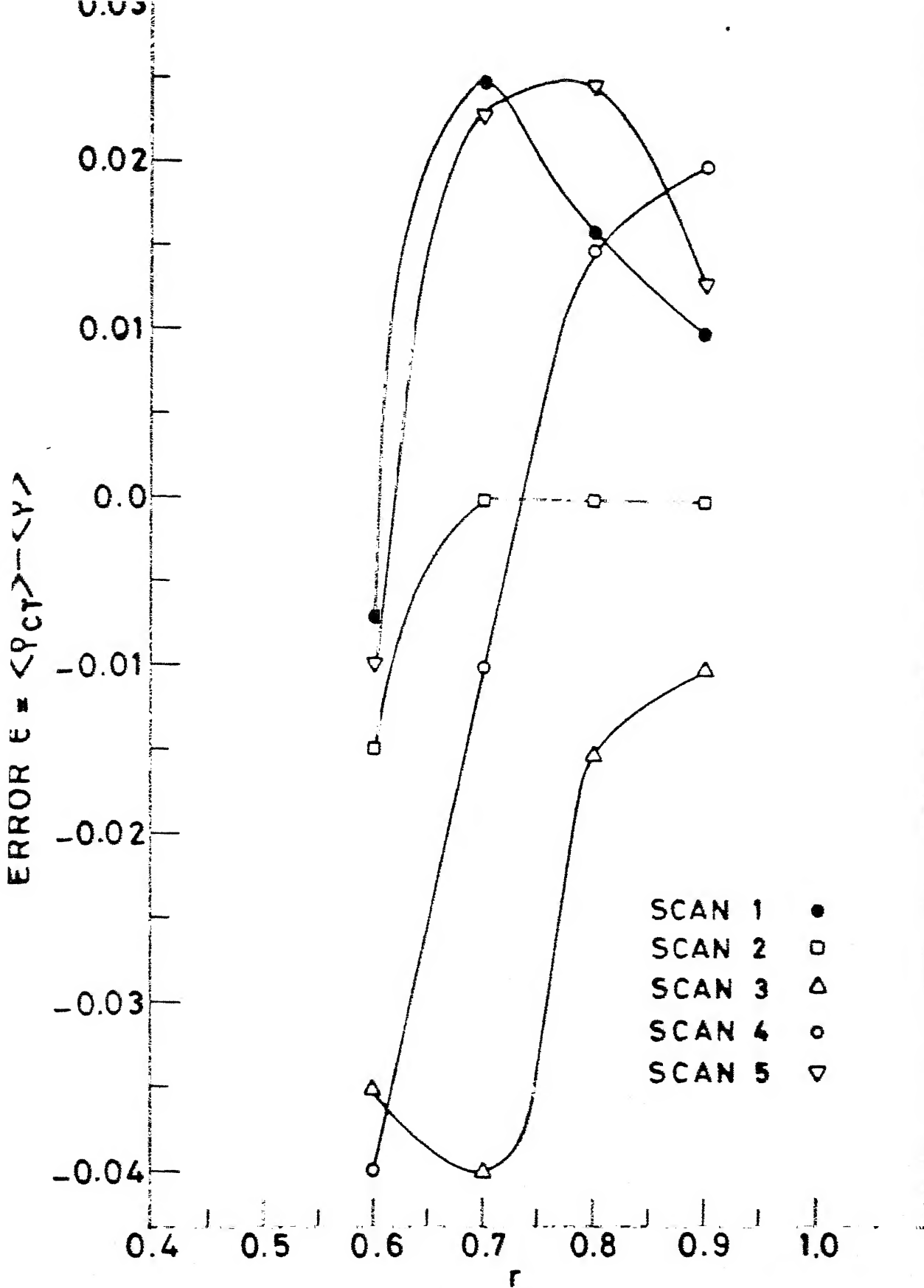
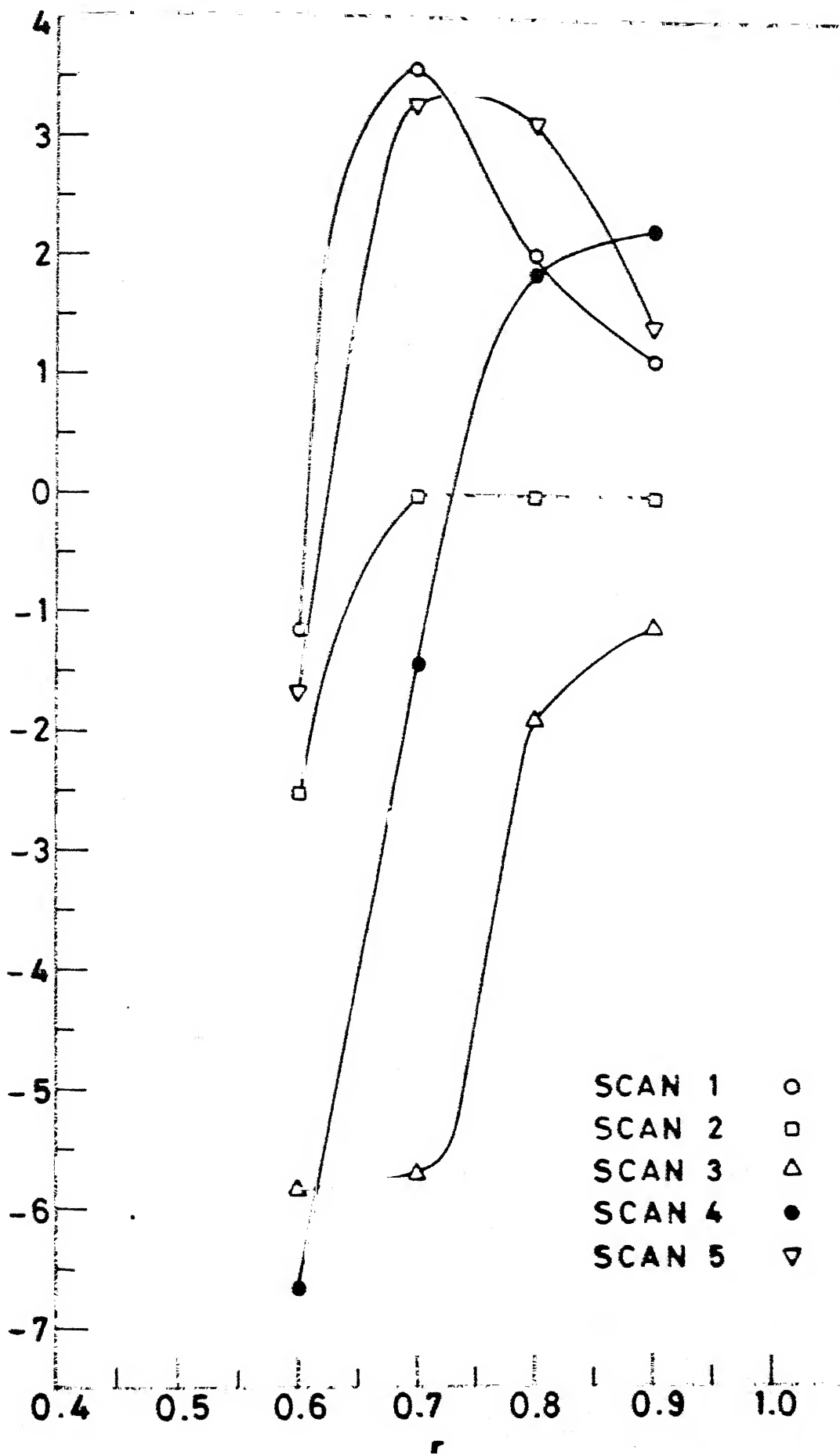


FIG.13 ERROR IN DENSITY MEASUREMENT FOR ALL SCANS.



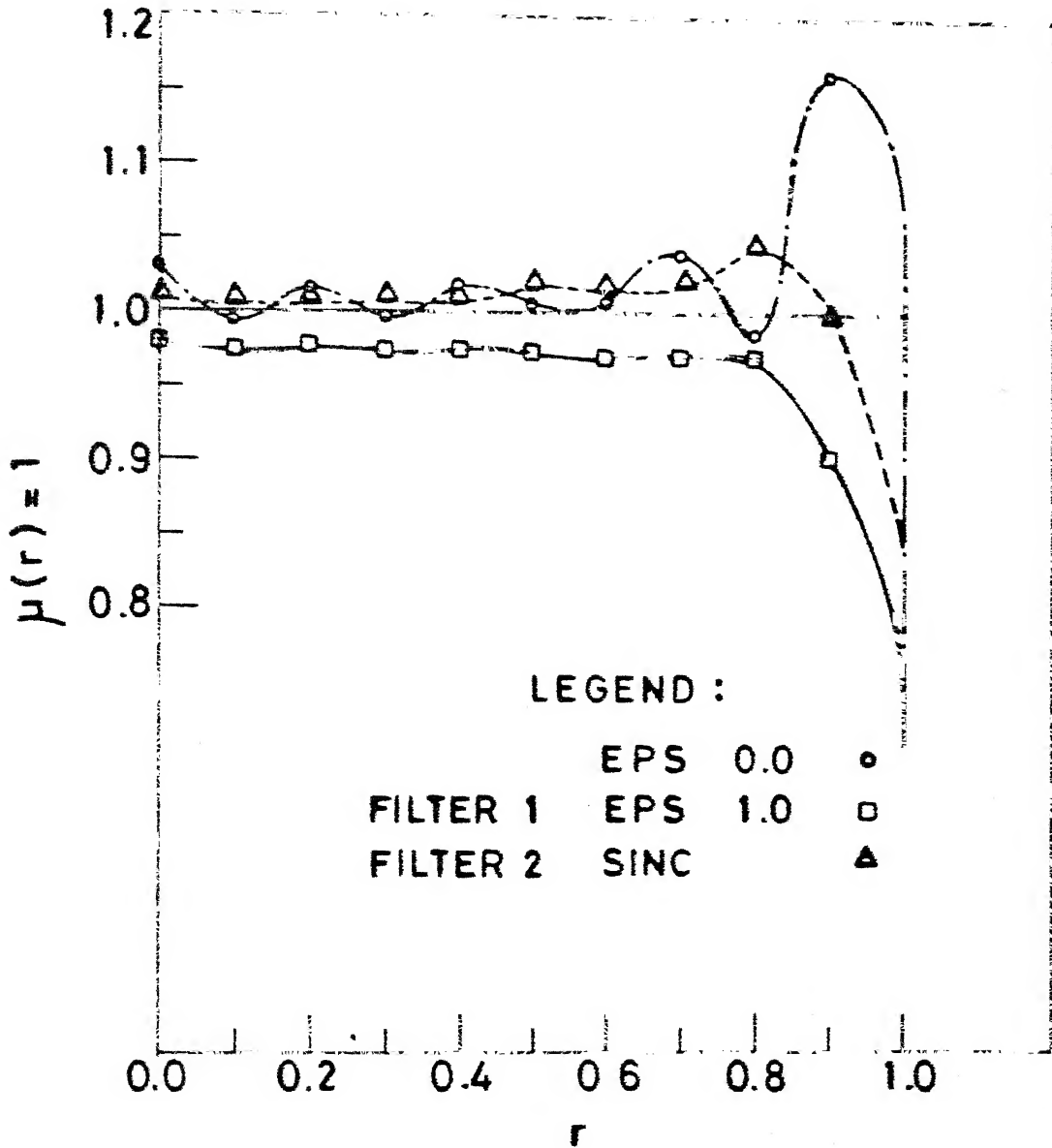


FIG.15 RESULTS WITH FILTERED VERSION.

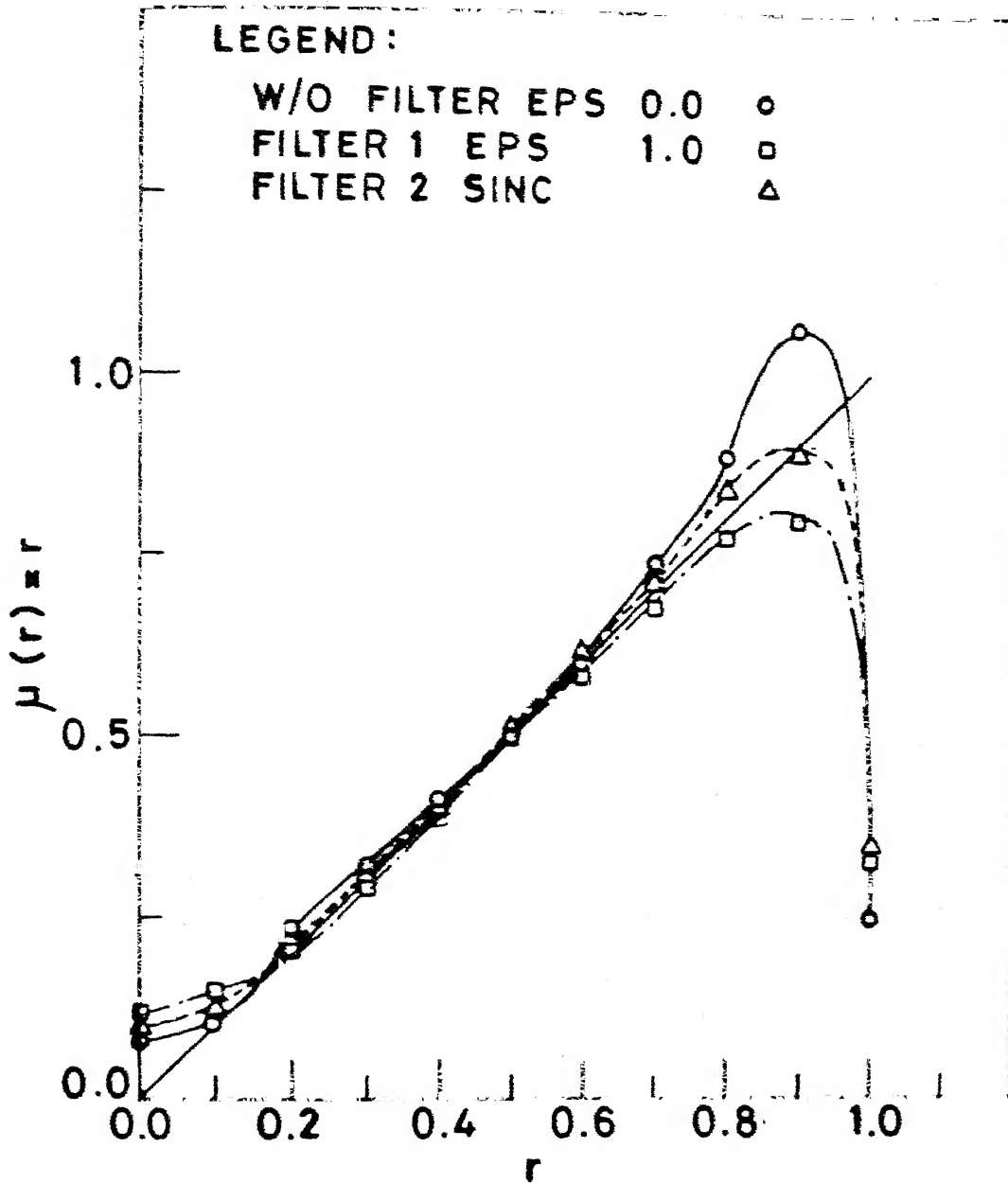


FIG. 16 RESULTS WITH FILTERED VERSION.

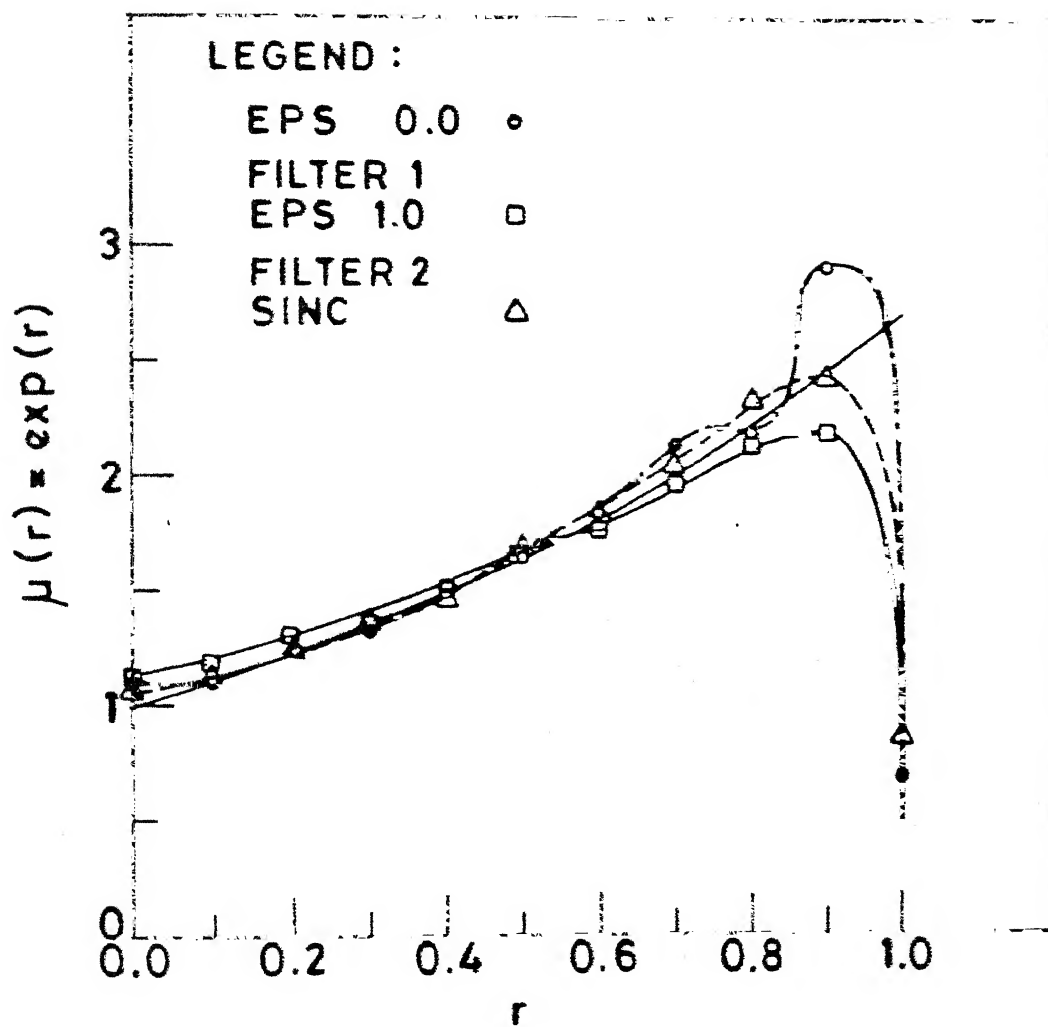


FIG.17 RESULTS WITH FILTERED VERSION.

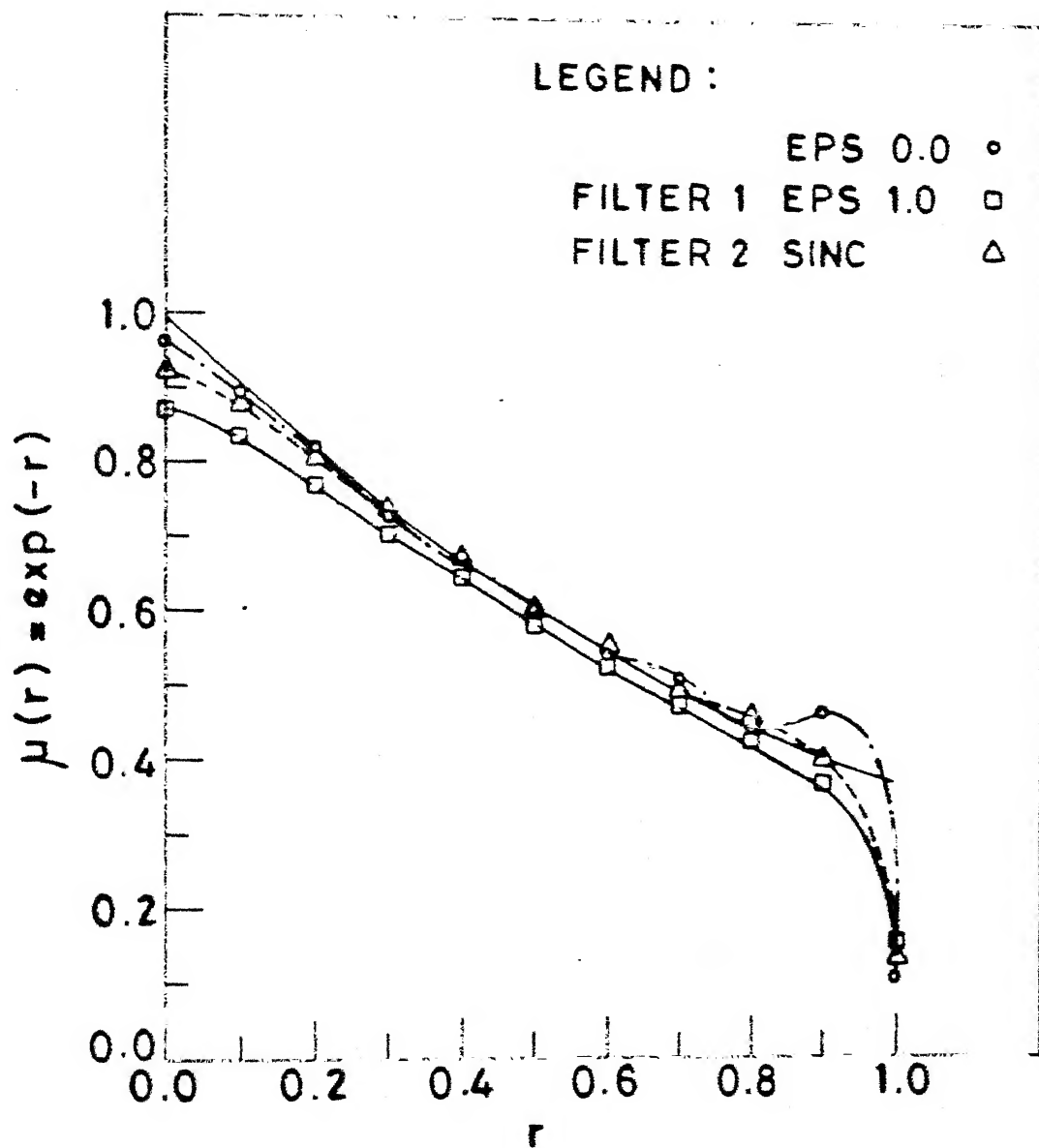


FIG.18 RESULTS WITH FILTERED VERSION.

CHAPTER 4

DISCUSSION

The proposed algorithm, using the simplified Radon Inversion Formulae, has been demonstrated to be applicable to radially symmetric two-phase flows. The unfiltered version ($\text{EPS} = 0.0$), incorporating a unit step cut-off in the Fourier space, gives results comparable to the more general methods of the same type [13].

However, we observe that the unfiltered version is prone to slight overshoots/undershoots, while the filtered versions are relatively free of the aforesaid oscillations. The results with the linear and sinc filters show a reasonable improvement in the reconstructed values (See Figs.15-18). These filtered versions are also comparable to the general tomographic methods involving the same filter-types [13].

The bubbly flow projection data processed by this algorithm was in the range of

$$0.6 \leq \langle \rho \rangle \leq 0.9 \text{ g/cm}^3$$

which is equivalent to,

$$0.1 \leq \langle \alpha \rangle \leq 0.4 ,$$

where α denotes the void fraction for the air water flow systems. The values of $\langle \rho_{CT} \rangle$ are within $\pm 0.03 \text{ g/cm}^3$ of

the true values, $\langle \rho \rangle$. This magnitude of the deviation compares favourably compared to the other existing methods of measuring density/void fraction in two phase flows.

We also note that the results reported here are comparable to the other algorithms developed [7,12] for radially symmetric situations.

Scan No. 1

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR	2869	2063	2084	2315	2404	2447	2485	2490	2516	2528	2534	2532	2527	2522	2526	2513	2492	2471	2449	2398	2308	2149	1743	2834	2842
46% VOID	2875	2109	1573	1518	1483	1448	1435	1434	1443	1456	1465	1464	1470	1469	1463	1466	1456	1423	1452	1464	1482	1541	1703	2830	2832
40% VOID	2826	2100	1552	1469	1402	1380	1351	1339	1332	1345	1349	1348	1342	1348	1361	1345	1357	1342	1364	1384	1422	1497	1667	2835	2846
30% VOID	2857	2061	1517	1377	1278	1218	1176	1160	1146	1130	1133	1126	1136	1139	1141	1158	1171	1176	1209	1269	1314	1430	1646	2835	2837
20% VOID	2864	2074	1500	1331	1218	1133	1079	1055	1018	1010	1002	1001	999	1005	1018	1023	1048	1077	1125	1198	1261	1390	1630	2824	2819
10% VOID	2879	2035	1505	1308	1173	1082	1005	958	913	894	884	876	894	894	902	923	952	991	1041	1127	1218	1370	1632	2800	2815
WATER	2885	2062	1495	1266	1106	996	913	858	811	781	758	748	740	745	765	790	826	877	951	1044	1153	1359	1663	2850	2835
WALNUT (ρ=0.732)	2853	2082	1742	1565	1449	1392	1335	1265	1206	1159	1141	1121	1126	1108	1113	1132	1161	1207	1282	1365	1458	1619	1705	2846	2846
PINE (ρ=0.41)	2858	2060	1773	1863	1800	1754	1705	1678	1639	1616	1601	1599	1595	1594	1610	1621	1645	1669	1715	1750	1782	1856	1710	2834	2827

[illegible]

APPENDIX B

TABLE-B1

(Results for Air, Pine, Walnut and Water)

Case	$\langle \rho \rangle, \text{ g/cm}^3$	$\langle \text{CTN} \rangle$
Air	0.000	0.000
Pine	0.410	0.081
Walnut	0.732	0.146
Water	1.000	0.212

TABLE-B2

(Reconstructed Densities)

$\langle \rho \rangle \text{ g/cm}^3$	0.6	0.7	0.8	0.9
Scan No.				
1	0.59	0.72	0.82	0.91
2	0.58	0.70	0.80	0.90
3	0.57	0.67	0.78	0.89
4	0.56	0.69	0.81	0.92
5	0.59	0.72	0.82	0.91

TABLE-B3
(Reconstructed <CTNs>)

Scan No.	$\langle \rho \rangle \text{ g/cm}^3$			
	0.6	0.7	0.8	0.9
1	0.1165	0.1443	0.1638	0.1829
2	0.1150	0.1383	0.1601	0.1808
3	0.1114	0.1310	0.1572	0.1792
4	0.1097	0.1370	0.1610	0.1847
5	0.1157	0.1445	0.1653	0.1843

RESULTS FOR FUN = 1

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	1.033052	-0.033052
0.100	1.990	0.100	1.000	0.995444	-0.004556
0.200	1.960	0.200	1.000	1.017349	-0.017349
0.300	1.908	0.300	1.000	0.995643	-0.004357
0.400	1.833	0.400	1.000	1.017732	-0.017732
0.500	1.732	0.500	1.000	1.005409	-0.005409
0.600	1.600	0.600	1.000	1.005603	-0.005603
0.700	1.428	0.700	1.000	1.040422	-0.040422
0.800	1.200	0.800	1.000	0.988325	-0.011675
0.900	0.872	0.900	1.000	1.167412	-0.167412
1.000	0.000	1.000	1.000	0.270418	-0.729582

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	0.982484	0.017515
0.100	1.990	0.100	1.000	0.978126	0.021874
0.200	1.950	0.200	1.000	0.979857	0.020143
0.300	1.903	0.300	1.000	0.978210	0.021790
0.400	1.833	0.400	1.000	0.977122	0.022878
0.500	1.732	0.500	1.000	0.977493	0.022507
0.600	1.600	0.600	1.000	0.972692	0.027308
0.700	1.428	0.700	1.000	0.973752	0.026248
0.800	1.200	0.800	1.000	0.972126	0.027874
0.900	0.872	0.900	1.000	0.906990	0.093010
1.000	0.000	1.000	1.000	0.372651	0.627349

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	1.009075	-0.009075
0.100	1.990	0.100	1.000	1.004987	-0.004987
0.200	1.950	0.200	1.000	1.007308	-0.007308
0.300	1.903	0.300	1.000	1.006914	-0.006914
0.400	1.833	0.400	1.000	1.007557	-0.007557
0.500	1.732	0.500	1.000	1.010950	-0.010950
0.600	1.600	0.600	1.000	1.010897	-0.010897
0.700	1.428	0.700	1.000	1.019755	-0.019755
0.800	1.200	0.800	1.000	1.043319	-0.043319
0.900	0.872	0.900	1.000	0.996265	0.003734
1.000	0.000	1.000	1.000	0.358192	0.641808

RESULTS FOR FUN = π

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.0000	0.0000	0.0000	0.078407	-0.078407
0.100	1.0025	0.1000	0.1000	0.103694	-0.0033094
0.200	1.0071	0.2000	0.2000	0.221228	-0.0021228
0.300	1.1123	0.3000	0.3000	0.298952	-0.0001048
0.400	1.1167	0.4000	0.4000	0.419104	-0.0019104
0.500	1.1195	0.5000	0.5000	0.507586	-0.007586
0.600	1.1186	0.6000	0.6000	0.606550	-0.006550
0.700	1.1153	0.7000	0.7000	0.741505	-0.041505
0.800	1.0444	0.8000	0.8000	0.789629	-0.010371
0.900	0.814	0.9000	0.9000	1.066993	-0.166993
1.000	0.0000	1.0000	1.0000	0.254635	0.745365

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.0000	0.0000	0.0000	0.124132	-0.124132
0.100	1.0025	0.1000	0.1000	0.157358	-0.057358
0.200	1.0071	0.2000	0.2000	0.236552	-0.036552
0.300	1.1123	0.3000	0.3000	0.322136	-0.022136
0.400	1.1167	0.4000	0.4000	0.411920	-0.011920
0.500	1.1195	0.5000	0.5000	0.504830	-0.004830
0.600	1.1186	0.6000	0.6000	0.593349	-0.003551
0.700	1.1153	0.7000	0.7000	0.687810	-0.012190
0.800	1.0444	0.8000	0.8000	0.779142	-0.020858
0.900	0.814	0.9000	0.9000	0.803276	-0.006724
1.000	0.0000	1.0000	1.0000	0.338295	0.661705

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.0000	0.0000	0.0000	0.086652	-0.086652
0.100	1.0025	0.1000	0.1000	0.126537	-0.026537
0.200	1.0071	0.2000	0.2000	0.217543	-0.017543
0.300	1.1123	0.3000	0.3000	0.313544	-0.013544
0.400	1.1167	0.4000	0.4000	0.412496	-0.012496
0.500	1.1195	0.5000	0.5000	0.514872	-0.014872
0.600	1.1186	0.6000	0.6000	0.614153	-0.014153
0.700	1.1153	0.7000	0.7000	0.722440	-0.022440
0.800	1.0444	0.8000	0.8000	0.846333	-0.046333
0.900	0.814	0.9000	0.9000	0.896221	-0.003779
1.000	0.0000	1.0000	1.0000	0.343932	0.656068

RESULTS FOR FUN = EXP(π)

P(L) = DATA

WITHOUT L-VAL	FILTERED P(L)	VERSION RSMALL	F(R)	INT-VAL	ERROR
0.000	3.437	0.000	1.000	1.138516	-0.138516
0.100	3.457	0.100	1.105	1.102060	-0.003111
0.200	3.491	0.200	1.221	1.274623	-0.053220
0.300	3.519	0.300	1.350	1.343425	0.006434
0.400	3.527	0.400	1.492	1.543105	-0.051280
0.500	3.496	0.500	1.649	1.668565	-0.019844
0.600	3.404	0.600	1.822	1.840291	-0.018172
0.700	3.215	0.700	2.014	2.127485	-0.113733
0.800	2.859	0.800	2.226	2.198562	0.026979
0.900	2.219	0.900	2.460	2.915379	-0.455770
1.000	0.000	1.000	2.718	0.693351	2.024931

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	3.437	0.000	1.000	1.130545	-0.130545
0.100	3.457	0.100	1.105	1.115172	-0.005655
0.200	3.491	0.200	1.221	1.250079	-0.038576
0.300	3.519	0.300	1.350	1.371043	-0.021184
0.400	3.527	0.400	1.492	1.500060	-0.008220
0.500	3.496	0.500	1.649	1.648885	-0.000015
0.600	3.404	0.600	1.822	1.799902	0.022217
0.700	3.215	0.700	2.014	1.933251	0.030491
0.800	2.859	0.800	2.226	2.176438	0.049193
0.900	2.219	0.900	2.460	2.205919	0.252588
1.000	0.000	1.000	2.718	0.927383	1.790599

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	3.437	0.000	1.000	1.106184	-0.105184
0.100	3.457	0.100	1.105	1.144564	-0.039393
0.200	3.491	0.200	1.221	1.256031	-0.034528
0.300	3.519	0.300	1.350	1.380233	-0.030374
0.400	3.527	0.400	1.492	1.522628	-0.030804
0.500	3.496	0.500	1.649	1.688138	-0.039417
0.600	3.404	0.600	1.822	1.851260	-0.039141
0.700	3.215	0.700	2.014	2.076739	-0.052985
0.800	2.859	0.800	2.226	2.354704	-0.429154
0.900	2.219	0.900	2.460	2.453574	0.005029
1.000	0.000	1.000	2.718	0.937485	1.780795

RESULTS FOR FUN = exp(-x)

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
.000	1.264	0.000	1.000	0.969042	0.030958
.100	1.234	0.100	0.905	0.896158	0.008679
.200	1.159	0.200	0.819	0.822768	-0.004037
.300	1.084	0.300	0.741	0.737237	-0.003581
.400	0.989	0.400	0.670	0.676513	-0.006193
.500	0.879	0.500	0.607	0.607694	-0.001164
.600	0.754	0.600	0.549	0.550756	-0.001944
.700	0.640	0.700	0.497	0.511359	-0.014774
.800	0.530	0.800	0.449	0.444774	-0.004555
.900	0.343	0.900	0.407	0.468666	-0.062096
1.000	0.000	1.000	0.368	0.105423	0.262456

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
.000	1.254	0.000	1.000	0.874447	0.125553
.100	1.234	0.100	0.905	0.839713	0.055123
.200	1.159	0.200	0.819	0.775718	0.043012
.300	1.084	0.300	0.741	0.708550	0.032269
.400	0.989	0.400	0.670	0.644976	0.025344
.500	0.879	0.500	0.607	0.586535	0.019396
.600	0.754	0.600	0.549	0.531310	0.017501
.700	0.640	0.700	0.497	0.482955	0.013551
.800	0.530	0.800	0.449	0.438275	0.011054
.900	0.343	0.900	0.407	0.375638	0.030931
1.000	0.000	1.000	0.368	0.151080	0.216799

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
.000	1.254	0.000	1.000	0.929526	0.070474
.100	1.234	0.100	0.905	0.888782	0.016055
.200	1.159	0.200	0.819	0.814566	0.004165
.300	1.084	0.300	0.741	0.739895	0.000923
.400	0.989	0.400	0.670	0.671094	-0.000774
.500	0.879	0.500	0.607	0.609357	-0.002826
.600	0.754	0.600	0.549	0.552115	-0.003304
.700	0.640	0.700	0.497	0.503478	-0.006833
.800	0.530	0.800	0.449	0.465285	-0.015955
.900	0.343	0.900	0.407	0.405923	0.000547
1.000	0.000	1.000	0.368	0.144746	0.223133

FUNCTION= 1, 0,
RESULTS WITHOUT FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	0.873292	0.126708
0.100	0.775	0.100	1.000	1.094587	-0.094587
0.200	0.693	0.200	1.000	0.974604	0.025396
0.300	0.529	0.300	1.000	1.188855	-0.188855
0.400	0.000	0.400	0.000	0.264220	-0.264220
0.500	0.000	0.500	0.000	-0.021588	0.021588
0.600	0.000	0.600	0.000	-0.005726	0.005726
0.700	0.000	0.700	0.000	0.012578	-0.012578
0.800	0.000	0.800	0.000	-0.012079	0.012079
0.900	0.000	0.900	0.000	0.008302	-0.008302
1.000	0.000	1.000	0.000	-0.003474	0.003474

RESULTS WITH LINEAR FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	0.950976	0.049024
0.100	0.775	0.100	1.000	0.960556	0.039444
0.200	0.693	0.200	1.000	0.958434	0.041566
0.300	0.529	0.300	1.000	0.886979	0.113021
0.400	0.000	0.400	0.000	0.340691	-0.340691
0.500	0.000	0.500	0.000	0.034996	-0.034996
0.600	0.000	0.600	0.000	0.019467	-0.019467
0.700	0.000	0.700	0.000	0.007950	-0.007950
0.800	0.000	0.800	0.000	0.005277	-0.005277
0.900	0.000	0.900	0.000	0.003728	-0.003728
1.000	0.000	1.000	0.000	0.002129	-0.002129

RESULTS WITH SINC FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	1.021504	-0.021504
0.100	0.775	0.100	1.000	1.032216	-0.032216
0.200	0.693	0.200	1.000	1.056784	-0.056784
0.300	0.529	0.300	1.000	0.998335	0.001665
0.400	0.000	0.400	0.000	0.350906	-0.350906
0.500	0.000	0.500	0.000	-0.002565	0.002565
0.600	0.000	0.600	0.000	0.002971	-0.002971
0.700	0.000	0.700	0.000	-0.000975	0.000975
0.800	0.000	0.800	0.000	0.000096	-0.000096
0.900	0.000	0.900	0.000	0.000230	-0.000230
1.000	0.000	1.000	0.000	-0.000278	0.000278

FUNCTION= 0, 1, 0,
RESULTS WITHOUT FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	0.004273	-0.004273
0.100	0.428	0.100	0.000	-0.168250	0.168250
0.200	0.693	0.200	1.000	0.752535	0.247465
0.300	0.529	0.300	1.000	1.189902	-0.189902
0.400	0.000	0.400	0.000	0.277719	-0.277719
0.500	0.000	0.500	0.000	-0.032493	0.032493
0.600	0.000	0.600	0.000	-0.000701	0.000701
0.700	0.000	0.700	0.000	0.012591	-0.012591
0.800	0.000	0.800	0.000	-0.015140	0.015140
0.900	0.000	0.900	0.000	0.012185	-0.012185
1.000	0.000	1.000	0.000	-0.006480	0.006480

RESULTS WITH LINEAR FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	-0.031784	0.031784
0.100	0.428	0.100	0.000	0.111237	-0.111237
0.200	0.693	0.200	1.000	0.662904	0.337096
0.300	0.529	0.300	1.000	0.865200	0.134800
0.400	0.000	0.400	0.000	0.329495	-0.329495
0.500	0.000	0.500	0.000	0.030962	-0.030962
0.600	0.000	0.600	0.000	0.017311	-0.017311
0.700	0.000	0.700	0.000	0.006223	-0.006223
0.800	0.000	0.800	0.000	0.004505	-0.004505
0.900	0.000	0.900	0.000	0.002997	-0.002997
1.000	0.000	1.000	0.000	0.001633	-0.001633

RESULTS WITH SINC FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	-0.148578	0.148578
0.100	0.428	0.100	0.000	0.034873	-0.034873
0.200	0.693	0.200	1.000	0.734247	0.265753
0.300	0.529	0.300	1.000	1.000623	-0.000623
0.400	0.000	0.400	0.000	0.348814	-0.348814
0.500	0.000	0.500	0.000	-0.002141	0.002141
0.600	0.000	0.600	0.000	0.003125	-0.003125
0.700	0.000	0.700	0.000	-0.001235	0.001235
0.800	0.000	0.800	0.000	0.000282	-0.000282
0.900	0.000	0.900	0.000	0.000154	-0.000154
1.000	0.000	1.000	0.000	-0.000290	0.000290

DELTA -FUNCTION
WITHOUT FILTERED VERSION

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	6.371062
0.100	0.00	0.10	1.042347
0.200	0.00	0.20	0.012490
0.300	0.00	0.30	-0.091487
0.400	0.00	0.40	0.079255
0.500	0.00	0.50	-0.045604
0.600	0.00	0.60	0.012038
0.700	0.00	0.70	0.012054
0.800	0.00	0.80	-0.023465
0.900	0.00	0.90	0.023228
1.000	0.00	1.00	-0.015080

DELTA -FUNCTION
RESULTS WITH LINEAR FILTER

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	2.692080
0.100	0.00	0.10	1.087312
0.200	0.00	0.20	0.030145
0.300	0.00	0.30	0.026919
0.400	0.00	0.40	0.005704
0.500	0.00	0.50	0.002051
0.600	0.00	0.60	0.003769
0.700	0.00	0.70	-0.000012
0.800	0.00	0.80	0.001404
0.900	0.00	0.90	0.000579
1.000	0.00	1.00	0.000110

DELTA -FUNCTION
RESULTS WITH SINC FILTER

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	3.278601
0.100	0.00	0.10	1.307053
0.200	0.00	0.20	-0.020768
0.300	0.00	0.30	0.011097
0.400	0.00	0.40	-0.002024
0.500	0.00	0.50	-0.001107
0.600	0.00	0.60	0.001695
0.700	0.00	0.70	-0.001285
0.800	0.00	0.80	0.000619
0.900	0.00	0.90	-0.000047
1.000	0.00	1.00	-0.000292

RESULTS FOR AIR

Q-VAL	DATA	RSMALL	VALUE	VALINT(I)
0.000	7.931736	0.000	0.343073	0.343073
0.305	7.932308	0.305	0.765102	0.765102
0.610	7.931302	0.610	0.888887	0.888887
0.914	7.920233	0.914	0.839703	0.839703
1.216	7.920841	1.216	0.717628	0.717628
1.515	7.912378	1.515	0.690321	0.690321
1.812	7.903135	1.812	0.853521	0.853521
2.105	7.782390	2.105	1.038153	1.038153
2.394	7.714137	2.394	0.990076	0.990076
2.679	7.672758	2.679	0.962430	0.962430
2.958	7.463353	2.958	1.335429	1.335429
3.232	7.949444	3.232	1.890475	1.890475
3.500	7.952253	3.500	4.785457	4.785457

AREA AVERAGE=0.000000

RESULTS FOR WATER

Q-VAL	DATA	RSMALL	VALUE	VALINT(I)
0.000	6.606650	0.000	5.502898	0.343073
0.305	6.613394	0.305	0.330819	0.765102
0.610	6.639876	0.610	0.657222	0.888887
0.914	6.672033	0.914	0.659477	0.839703
1.216	6.716595	1.216	0.528999	0.717628
1.515	6.776507	1.515	0.474123	0.690321
1.812	6.857514	1.812	0.628021	0.853521
2.105	6.950815	2.105	0.832620	1.038153
2.394	7.050123	2.394	0.780065	0.990076
2.679	7.207866	2.679	0.747661	0.962430
2.958	7.416378	2.958	1.279630	1.335429
3.232	7.955074	3.232	1.898424	1.890475
3.500	7.949797	3.500	4.779936	4.785457

AREA AVERAGE=0.212267

RESULTS FOR WALNUT

L-VAL	DATA	RSVAL	VALUE	VALINT(I)	VALACT(I)
0.000	7.026427	0.000	-0.071444	0.013646	0.085590
0.305	7.010312	0.305	0.643460	0.773350	0.129489
0.610	7.014814	0.610	0.715929	0.859481	0.143552
0.914	7.034741	0.914	0.673239	0.819705	0.146466
1.216	7.057037	1.216	0.579101	0.727867	0.148766
1.515	7.095823	1.515	0.502908	0.714355	0.151748
1.812	7.156177	1.812	0.704992	0.853479	0.148496
2.105	7.218910	2.105	0.867227	1.012520	0.145233
2.394	7.284821	2.394	0.840427	0.990897	0.150470
2.670	7.389564	2.670	0.861379	0.996381	0.135001
2.958	7.441320	2.958	1.263031	1.289660	0.026629
3.232	7.953670	3.232	1.016095	1.917099	0.000414
3.500	7.953670	3.500	4.681117	4.680028	-0.001089

AREA AVERAGE= 0.146203

RESULTS FOR PINE

L-VAL	DATA	RSVAL	VALUE	VALINT(I)	VALACT(I)
0.000	7.374629	0.000	-0.066580	0.013646	0.080231
0.305	7.374602	0.305	0.692213	0.773350	0.081136
0.610	7.383989	0.610	0.770305	0.859481	0.080087
0.914	7.390799	0.914	0.737285	0.819705	0.082420
1.216	7.405406	1.216	0.648432	0.727867	0.079384
1.515	7.419980	1.515	0.632811	0.714355	0.081545
1.812	7.47168	1.812	0.773668	0.853479	0.079809
2.105	7.467371	2.105	0.931737	1.012520	0.080783
2.394	7.485492	2.394	0.903411	0.990897	0.087486
2.670	7.526179	2.670	0.927431	0.996381	0.069950
2.958	7.44249	2.958	1.273474	1.289660	0.016186
3.232	7.049444	3.232	1.016761	1.917099	0.000338
3.500	7.046971	3.500	4.677223	4.680028	0.002805

AREA AVERAGE= 0.080758

L-VAL	P(L)	RS-MALL	VALUE	VALINT(I)	VALACT(I)
0.050	6.735706	0.000	1534378	0.0133559	0.1670248
0.050	6.735706	0.000	1506832	0.0773358	0.1649189
0.050	6.735706	0.000	6847222	0.0855970	0.1746425
0.050	6.735706	0.000	6553522	0.0817357	0.1749345
0.050	6.735706	0.000	5388250	0.0714337	0.1755317
0.050	6.735706	0.000	5679917	0.0855970	0.1854829
0.050	6.735706	0.000	6305825	0.0100339	0.1803660
0.050	6.735706	0.000	8065325	0.0995593	0.1997999
0.050	6.735706	0.000	203778	0.0281702	0.049115
0.050	6.735706	0.000	674545	0.0168002	0.007321
0.050	6.735706	0.000			0.005483

L-VAL	P(L)	RS-MALL	VALUE	VALINT(I)	VALACT(I)
0.050	6.735706	0.000	151730	0.0133559	0.165376
0.050	6.735706	0.000	128169	0.0773358	0.145183
0.050	6.735706	0.000	718358	0.0855970	0.141072
0.050	6.735706	0.000	670473	0.0817357	0.151349
0.050	6.735706	0.000	558533	0.0714337	0.151881
0.050	6.735706	0.000	599381	0.0855970	0.156444
0.050	6.735706	0.000	851483	0.0100339	0.160572
0.050	6.735706	0.000	804866	0.0995593	0.176067
0.050	6.735706	0.000	385667	0.0281702	0.191971
0.050	6.735706	0.000	67583	0.0168002	0.150453
0.050	6.735706	0.000			0.004191

L-VAL	P(L)	RS-MALL	VALUE	VALINT(I)	VALACT(I)
0.050	6.735706	0.000	151730	0.0133559	0.165376
0.050	6.735706	0.000	128169	0.0773358	0.145183
0.050	6.735706	0.000	718358	0.0855970	0.141072
0.050	6.735706	0.000	670473	0.0817357	0.151349
0.050	6.735706	0.000	558533	0.0714337	0.151881
0.050	6.735706	0.000	599381	0.0855970	0.156444
0.050	6.735706	0.000	851483	0.0100339	0.160572
0.050	6.735706	0.000	804866	0.0995593	0.176067
0.050	6.735706	0.000	385667	0.0281702	0.191971
0.050	6.735706	0.000	67583	0.0168002	0.150453
0.050	6.735706	0.000			0.004191

```

C *****
C THIS PROGRAM EVALUATES THE FUNCTION f(r) FOR DIFFERENT
C VALUES OF r BETWEEN 0.0 - 1.0 IN STEP OF 0.10
C INPUT DATA REQUIRED
C NDATA values of 1 and 6(1) FOR THE FUNCTIONS LIKE
C F(E)=1.0,1,exp(r),exp(-r) and for different flow lengths
C ARE GIVEN AS INPUT DATA FOR THIS PROGRAM
C *****
C COMMON/I1/NDATA,p(20),plval(20)
C COMMON/I6/EPSWP,RC,INDWR,IWRTYP,IAREA
C COMMON/I5/VALACT(15),R(15)
C DIMENSION PHATR(165),VALINT(15)
C
C PI=4.0*ATAN(1.0)
C
C NDATA - # OF DATA POINTS
C INDWR -=>1 NO WINDOW FN IS USED
C ->1 OR <1 WINDOW FN IS USED
C IWRTYP -=>2 SINC WINDOW USED
C ->2 OR <2 LINEAR WINDOW USED
C IAREA -GE 0 AREA CAL IS DONE & DATA IS STORED
C -< 0 NO AREA CAL IS DONE
C IATR -=>1 AIR DATA READ & BACKGROUND DATA/COUNTS STORED(AIR)
C -<1 OR >1 AREA CAL DONE
C
C READ INPUT DATA
C ACCEPT *,NDATA,INDWR,EPSWP,IWRTYP
C ACCEPT *,IAREA,IAIP
C
C IF(IAREA.LT.0)GO TO 30
C D=7.0
C DO 10 I=1,NDATA
C READ(24,*)P(I),PLVAL(I)
C P(I)=ALOG(P(I))
C 10 PLVAL(I)=0*SIN(PLVAL(I)*PI/180.0)
C IF(IAIR.EQ.1)GO TO 50
C DO 20 I=1,NDATA
C 20 READ(23,*)R(I),VALINT(I)
C GO TO 50
C
C 30 DO 40 I=1,NDATA
C 40 READ(24,*)P(I),PLVAL(I)
C SET VALUES OF WINDOW FN
C 50 RC=6.0
C N=160
C IF(IAREA.LT.0)GO TO 60
C WRITE(22,150)
C GO TO 90
C 60 WRITE(22,70)N

```

```

70  FORMAT(//20X,'N-VALUE= ',T6/)
    WRITE(22,80)
80  FORMAT(3X,'L-VAL',5X,'D(I)',5X,'RSMALL',5X,'F(R)',
    16X,'INT-VAL',5X,'ERROR'//)
C  LIMITS OF THE OUTERMOST INTERVAL A & B , STEP LENGTH H
90  A=0.0
    B=6.0
    H=(B-A)/N
    NN=N+1
C  SCORE PHATR FOR DIFFERENT STEPLENGTHS
    CALL PR(PHATR,NN,H)
    RSMALL=0.0
    DO 160 I=1,NDATA
    CONST=2.0*PI*RSMALL
C  INITIALISE I VALUE FOR A PARTICULAR I
    VALUE=0.0
C  INTEGRATION BY SIMPSON'S 1/3 RULE BEGINS
C  S17AEF IS A SYSTEM FUNCTION SUBPROGRAM WHICH EVALUATES
C  BESSEL'S FUNCTION AT A GIVEN POINT
C
    DO 100 J=2,N-2,2
    RI=A+H*FLOAT(J)
    CONJ=CONST*RI
    Y=S17AEF(CONJ,IFAIL)
    VALUE=VALUE+2.0*PI*PHATR(J+1)*Y*W(RI)
100  CONTINUE
C
    DO 110 J=1,N-1,2
    RI=A+H*FLOAT(J)
    CONJ=CONST*RI
    Y=S17AEF(CONJ,IFAIL)
    VALUE=VALUE+4.0*PI*PHATR(J+1)*Y*W(RI)
110  CONTINUE
C
    RI=B
    CONJ=CONST*RI
    Y=S17AEF(CONJ,IFAIL)
    VALUE=VALUE+PHATR(1)*RI*Y*W(RI)
C
    RI=A
    CONJ=CONST*RI
    Y=S17AEF(CONJ,IFAIL)
    VALUE=VALUE+PHATR(NN)*PI*Y*W(RI)
    VALUE=VALUE*H/3.0
C  F(R)=2.0*PI*VALUE
    VALUE=VALUE*2.0*PI
    PL=P(I)
C  EXACT VALUE OF THE FUNCTION FRR=f(r)
    FRR=1.0
    IF(RSMALL.GT.0.4)FRR=0.0
    IF(IAIR.EQ.1)FRR=1.0
C

```

```

C FOR VARIOUS TEST FNS REPLACE FPR IN THE ABOVE LINE BY
C THE CORRESPONDING FUNCTIONS
C FPR = 1.0 f(r)=1.0
C FPR = RSMALL f(r)=r ramp fn
C FPR = EXP(RSMALL) f(r)=exp(r) and
C FPR = EXP(-RSMALL) f(r)=exp(-r)
C
C IF(RSMALL.LT.0.2.OR.RSMALL.GT.0.4)FPR=0.0
C ESTIMATE THE ERROR (EXACT-RECONSTRUCTED)
120 FPR=FPR-VALUE
C
C IF(IAREA.LT.0)GO TO 130
C VALACT(I)=- (VALUE-VALINT(I))
C WRITE(22,170)PLVAL(I),P(I),RSMALL,VALUE,VALINT(I),VALACT(I)
C IF(IAP.EQ.1)WRITE(23,*)PLVAL(I),VALUE
C RSMALL=PLVAL(I+1)
C GO TO 160
130 WRITE(22,140),PLVAL(I),PI,RSMALL,FPR,VALUE,FPR
140 FORMAT(2X,4(F6.3,4X),2F10.6)
C RSMALL=RSMALL+0.1
150 FORMAT(3X,'L=VAL',9X,'P(L)',9X,'RSMALL',9X,'VALUE',
18X,'VALINT(I)',9X,'VALACT(I)')
160 CONTINUE
C
170 FORMAT(5X,F6.3,5X,F10.6,5X,F6.3,5X,3(F10.6,5X))
C IF(IAREA.LT.0)GO TO 180
C CALL AREA
180 CONTINUE
C STOP
C END
C
C *****
C THIS SUBROUTINE CALCULATES PHATR VALUES FOR DIFFERENT
C STEP-LENGTHS (R - VALUES) WHICH ARE USED IN THE MAIN
C PROGRAM FOR INTEGRATION
C *****
C
C SUBROUTINE PR(PHATR,N,P)
C
C COMMON/I2/R2PI
C COMMON/I1/NDATA,P(20),PLVAL(20)
C COMMON/I6/EPSWR,RC,INDWR,IWRTYP,IAREA
C DIMENSION PHATR(N)
C PI=4.0*ATAN(1.0)
C M=100
C IF(IAREA.GE.0)M=160
C LIMITS OF THE INTEGRAL PHAT(R)
C A=0.0
C B=1.0
C IF(IAREA.GE.0)B=3.5
C HPRIM=(B-A)/M

```



```

      R2PI=0.0
      DO 30 I=1,M
      VALUE=0.0
      DO 10 J=2,M-2,2
      SLV=A+HPRIM*FLOAT(J)
      VALUE=VALUE+2.0*PSLI(SLV)
10    CONTINUE
      DO 20 J=1,M-1,2
      SLV=A+HPRIM*FLOAT(J)
20    VALUE=VALUE+4.0*PSLI(SLV)
      SLV=A
      VALUE=VALUE+PSLI(SLV)
      SLV=R
      VALUE=VALUE+PSLI(SLV)
      VALUE=VALUE+HPRIM/3.0
      PRATR(I)=2.0*VALUE
      P=H*FLOAT(I)
      R2PI=R*2.0*PI
30    CONTINUE
      RETURN
      END

```

```

C *****
C THIS SUBROUTINE INTERPOLATES THE DATA P(1) FOR A GIVEN VALUE OF I
C *****
C      FUNCTION PSLI(SLI)
C
C      COMMON /I1/NDATA,P(20),PLVAL(20)
C      COMMON /I2/R2PI
C      COMMON I
C
C      IF(SLI.EQ.0.0)GO TO 30
C      DO 10 I=1,NDATA
C      IF(SLI.LE.PLVAL(I))GO TO 20
10    CONTINUE
20    LMAX=I
    LMIN=LMAX-1
    A=PLVAL(LMIN)
    B=PLVAL(LMAX)
    FCAL=(SLI-A)*(P(LMAX)-P(LMIN))/(B-A)
    PSLI=P(LMIN)+FCAL
    PSLI=PSLI*COS(R2PI*SLI)
    RETURN
30    PSLI=P(1)
    PSLI=PSLI*COS(R2PI*SLI)
    RETURN
    END
C *****

```

```

C THIS FUNCTION SUBPROGRAM RETURNS THE WINDOW FUNCTION VALUE
C FOR A GIVEN R VALUE
C THIS HAS TWO WINDOW FUNCTIONS
C 1) LINEAR AND 1) SINC
C IF INQWR.EQ.1, W RETURNS 1.0 ie W(R)=1.0 (NO WINDOW FN USED)
C IF INQWR.EQ.2, W(R) USES SINC FN, OTHERWISE USES LINEAR FN
C *****

```

```

      FUNCTION W(R)

```

```

      COMMON/16/EPSWR,RC,INQWR,INQWRTYP,TAREA

```

```

C WHEN W(R) IS NOT USED SET W(R)=1.0
      IF(INQWR.EQ.1)GO TO 20
      IF(INQWRTYP.EQ.2)GO TO 30
      IF(R.EQ.0.0)GO TO 10

```

```

      W=1.0-EPSWR*ABS(R)/RC
10      RETURN
20      W=1.0
      RETURN
30      IF(ABS(R).LE.0.1E-5)GO TO 20
      PI=4.0*ATAN(1.0)
      W=SIN(PI*ABS(R)/RC)/(PI*ABS(R)/RC)
      RETURN
      END

```

```

C *****
C THIS SUBROUTINE CALCULATES THE AVERAGE DENSITY FOR VARIOUS
C TYPES OF FLOWS
C *****

```

```

      SUBROUTINE AREA

```

```

      COMMON/15/VALACT(15),R(15)
      AREA=0.0

```

```

      DO 40 I=1,10
      VALUE=0.0
      NITRN=100
      A=0.0
      B=R(I)
      H=(B-A)/NITRN
      IF(A.EQ.B)GO TO 30
      DO 10 J=2,NITRN-2,2
      RS=A+H*FLOAT(J)
10      VALUE=VALUE+RS*FRS(RS)*2.0
      DO 20 J=1,NITRN-1,2
      RS=A+H*FLOAT(J)
20      VALUE=VALUE+RS*FRS(RS)*4.0
      VALUE=VALUE+A*FRS(A)

```

```

VALUE=(VALUE+R*FRS(B))*R/3.0
AREA=2.0*VALUE/(R*R)
30 CONTINUE
40 CONTINUE
50 WRITE(22,50)AREA,R(10)
FORMAT(//20X,'AREA AVERAGE=',F10.6,6X,'LIMITS 0.0-',F10.6)
RETURN
END

```

```

C *****
C THIS IS AN INTERPOLATION FUNCTION SUBPROGRAM USED IN AREA
C CALCULATIONS
C *****

```

```

FUNCTION FRS(X)

```

```

COMMON/15/VALACT(15),R(15)
IF(X.EQ.0.0)GO TO 30
DO 10 I=1,10
IF(X.LE.R(I))GO TO 20
10 CONTINUE
20 LMAX=I
LMIN=I-1
A=R(LMIN)
B=R(LMAX)
FCAL=(X-A)*(VALACT(LMAX)-VALACT(LMIN))/(B-A)
FRS=VALACT(LMIN)+FCAL
RETURN
30 FRS=VALACT(1)
RETURN
END

```

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